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Impact of a Non-Concentric Cylindrical Capacitor on the Characteristics of an RC DC-Driven Circuit

Haiduke Sarafian

University College, The Pennsylvania State University, York, USA
Email: has2@psu.edu

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Abstract

This report is a follow-up to the previously published article. The scope of the purely numeric study of the previous report is augmented and formulated analytically. The main objective is to analytically formulate the capacitance of a non-concentric, asymmetric, long cylindrical capacitor. It is proven that the capacitance is a function of the radii of the cylinders and the separation distance between the circular centers. It is also shown that the numeric conformal mapping of the previous work is closely related to the bipolar coordinate system. To accomplish the set goal, we utilized one of the popular Computer Algebra Systems (CAS), especially *Mathematica*. To underline the usefulness of the CAS, the analytic formulation, its associated numeric and graphical outputs are bundled into one file. It is shown under special conditions that the presented format approaches the well-known characteristic of the symmetric capacitor. Application-wise, the impact of the non-concentric cylindrical capacitor in an RC DC-driven electric circuit is reported. The report embodies the essential *Mathematica* codes, readily making the reproduction of the results attainable.

Keywords

Asymmetric Cylindrical Capacitor, Computer Algebra System, *Mathematica*, Bipolar Coordinate, RC Electric Circuit

1. Introduction

This report is a follow-up to the previously published article [1]. Symbolic formulation of the capacitance of traditional, commonly used, capacitors, such as parallel-plates, spherical, and cylindrical, is straightforward [2] [3]. Formulation of the mentioned cases encounters no conceptual and/or mathematical challenges. This is because these examples are geometrically symmetrized. In contrast, for instance, the analysis of an asymmetric cylindrical capacitor composed of parallel but off-centered cylinders conceptually and mathematically is challenging. Mainly because the Gauss law, which is the core conceptual vehicle (tool), is not applicable, recognizing this fundamental flaw, this report shows an alternative suitable method addressing the asymmetrical issue. The Conclusion segment of this report presents suggestions that apply the analysis to additional asymmetry cases.

In fact, two alternative methods: **2a** and **2c** have been considered. These methods differ from one another but are conducive to the same result.

The Analysis section is closed by reporting the impact of the finding on an RC DC-driven electric circuit.

This report is composed of three sections. In addition to the Introduction, Section 2 Formulation and Analysis, which is formed into four subsections, discusses the formulation. Section 2d embodies the CAS-based results. The last section, 3, is the Conclusions and Remarks, and a comment about the future plan.

The profile of the proposed problem is depicted in Figure 1.

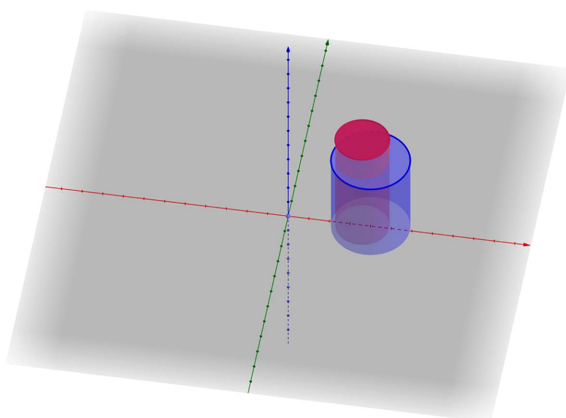

Figure 1. Depicts the 3-dimensional profile of an asymmetric cylindrical capacitor. It is composed of two different radii of parallel off-centered cylinders.

2. Formulation and Analysis

A coax cable is composed of two parallel circular cross-sectional cylinders with infinite length. Although it is straightforward to extend the issues of interest even if the space between the shells is filled with a dielectric, this article focuses on a variety of nontrivial problems without a dielectric. Here, we assume the cylinders are off-center, making the configuration asymmetric. And, because of the assumed "infinite" length of the cylinders, the issues of interest are confined to a two-dimensional analysis. This section is composed of four subsections addressing the problems of interest.

2a. Step1. Because of the asymmetry of the capacitor shown in Figure 1, we present the pieces (components) essentially needed to develop the final goal. First consider two geometrical points, $P_1(-a,0)$ and $P_2(a,0)$ shown in **Figure 2**.

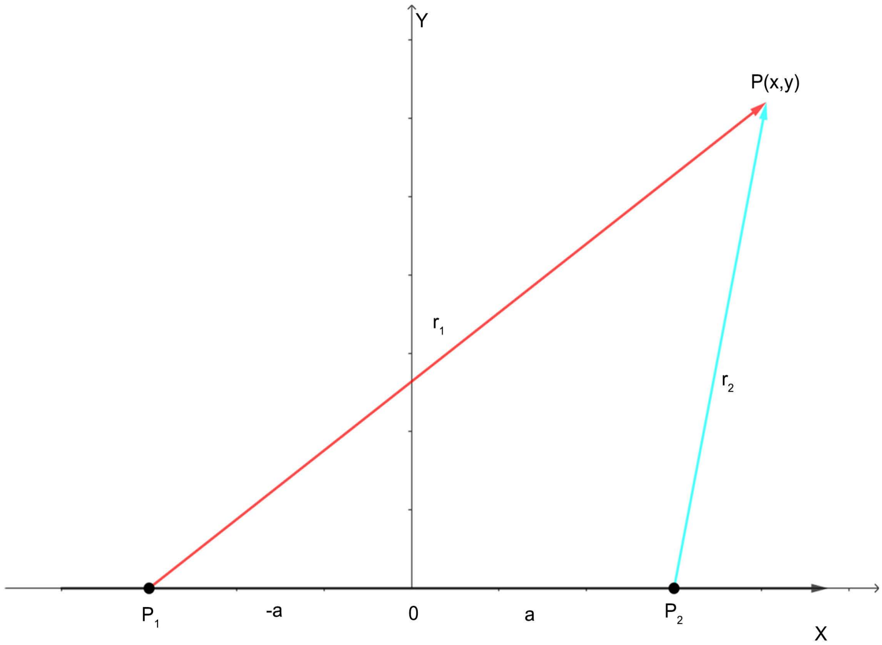

Figure 2. Two geometric points, $P_1(-a,0)$ and $P_2(a,0)$, and a third arbitrary point, $P(x,y)$, in Cartesian coordinates are shown.

The distance of an arbitrarily point, $P(x,y)$ from these two points are labeled r_1 and r_2 are: $r_1=\sqrt{(x+a)^2+y^2}$ and $r_2=\sqrt{(x-a)^2+y^2}$, respectfully. For the reasons explained later, we form the ratio, r_1/r_2, setting it equal to a constant value, c. In other words, we seek the locus of the points, $P(x,y)$, on the plane subject to the chosen constant value c. Manipulating the ratio for $c>1$, yields,

$$\left(x-a\frac{c^2+1}{c^2-1}\right)^2-a^2\left[\left(\frac{c^2+1}{c^2-1}\right)^2-1\right]+y^2=0, \tag{1}$$

Equation (1) is manipulated by yielding,

$$\left[x-a\frac{c^2+1}{c^2-1}\right]^2+y^2=a^2\left[\left(\frac{c^2+1}{c^2-1}\right)^2-1\right], \tag{2}$$

Equation (2) is abbreviated as,

$$(x-x_0)^2+y^2=R^2, \tag{3}$$

Where

$$x_0=a\frac{c^2+1}{c^2-1}, \text{ and } R=a\sqrt{\left(\frac{c^2+1}{c^2-1}\right)^2-1}. \tag{4}$$

Equation (3) is the equation of a circle centered at x_0 along the x-axis with a radius R.

2b. Step 2. By inserting two long, parallel, oppositely charged tiny rods, shown in red and the other one in blue, respectively, see **Figure 3**. The previous setting replaces the geometric problem with an electrostatic physics problem. The electrostatic potential at the last point, $P(x, y)$, in terms of two identical charged densities per unit length, can be expressed in terms of the separation distance between the rods and the charge densities [3]. The contour of the constant-valued potential traces circles. Two such circles associated with two-valued potentials are shown in **Figure 4**.

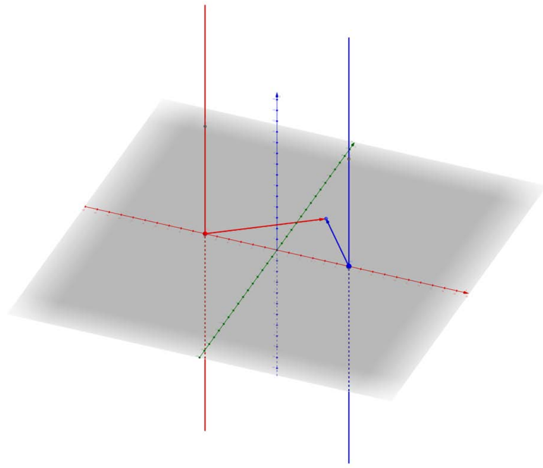

Figure 3. Two parallel oppositely charged lines, one in red and the other one in blue.

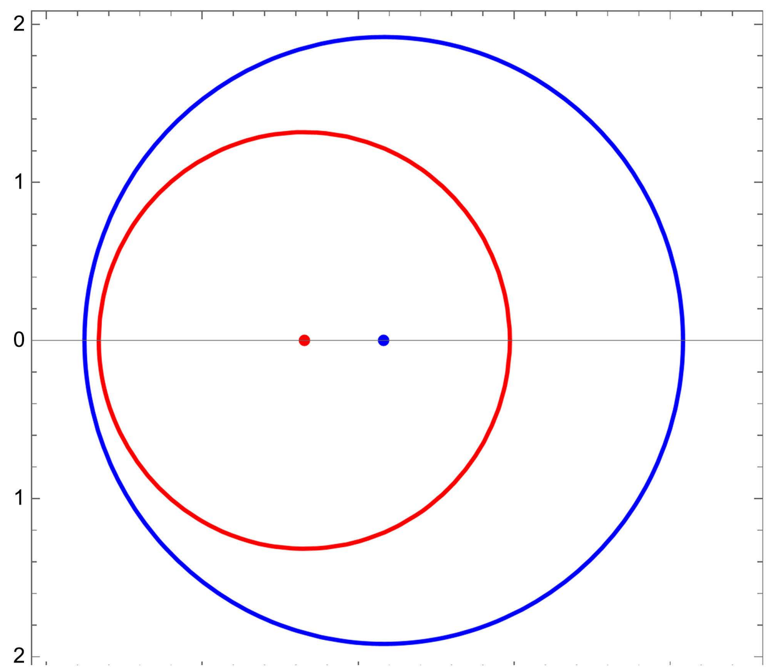

Figure 4. Two equipotential circles are associated with two different potentials.

The colors shown are indicative of different potentials. The colored dots are the center of the associated circles.

More specifically, by adapting the known [2] [3] information of electrostatic potential, φ of a long-charged line at a point distance r away from the line is $2k\lambda\ln(r)$, where the electrostatic coupling constant $k = 1/(4\pi\epsilon_0)$ and the charge density is $\lambda = \frac{q}{\ell}$. At the $P(x, y)$ point, the electrostatic potential emanating from the two lines is, $\varphi(x, y) = K[\ln(r_1) - \ln(r_2)]$, where $K = 2k\lambda$, or $\varphi = K\ln(r_1/r_2)$. In other words, the expression given in section 2a yields the relationship between geometry and physics, *i.e.*, is $r_1/r_2 = \exp(\Phi/K)$. Therefore, the mentioned constant value is $c = e^{\frac{\varphi}{K}}$.

Substituting $c = e^{\frac{\varphi}{K}}$ in (4) yields $x_0 = a\coth(\varphi/K)$ and $R = a\,\mathrm{csch}(\varphi/K)$. The circle Equation (3) in terms of physics specification becomes,

$$\left[x - a\coth\left(\frac{\varphi}{K}\right)\right]^2 + y^2 = \left[a\frac{1}{\sinh\left(\frac{\varphi}{K}\right)}\right]^2, \tag{5}$$

The value of the equi-potential yields to the coordinate of the circle along the $+x$-axis and its radius. Both characters have hyperbolic trig functions.

It is essential because the problem at hand involves two parallel charged lines; adjusting the separation between the lines results in their associated equi-potential surfaces resembling a non-concentric parallel cylindrical capacitor. It is valuable to recognize that a cylindrical capacitor composed of conducting cylinders, with assigned desired constant potentials, *i.e.*, constant equi-potential, surrounding the charged lines with constant potential surfaces, resembles cylindrical conducting surfaces.

2c. Bipolar Coordinate

In this sub-section, without an apparent relevant connection to the problem at hand, we review the bipolar coordinate system. The connection to the problem of interest becomes evident at the end of this sub-section.

In a complex plane, the complex bipolar coordinate is defined by [4],

$$x + \mathrm{i}y = \mathrm{i}a\cot\left(\frac{1}{2}\xi\right), \text{ where } \xi = \zeta + \mathrm{i}\eta, \tag{6}$$

The real and imaginary terms of (6) are yield,

$$\frac{x}{a} = \frac{\sinh(\eta)}{\cosh(\eta) - \cos(\xi)}, \tag{6a}$$

$$\frac{y}{a} = \frac{\sin(\xi)}{\cosh(\eta) - \cos(\xi)}, \tag{6b}$$

Equations (6a) and (6b) after lengthy algebraic manipulation yield,

$$[x - a\coth(\eta)]^2 + y^2 = a^2\frac{1}{\sinh^2(\eta)}, \tag{7}$$

Equation (7) depends only on the imaginary component of the argument of (6) and is expressed in terms of parametric hypergeometric trigonometric functions.

Equation (7) is the equation of a circle centered at $x_0 = a\coth(\eta)$ along the $+x$-axis with radius $R = a \cdot 1/\sinh(\eta)$. Although this is an obvious observation, when it is compared to (5) in Section 2b, which is formulated with physics insight, it reveals the hidden connection between the problem at hand and the bipolar coordinate system! In other words, the parameter η in (6) is the equipotential φ of the issue at hand!

For the numeric computation, it is insightful to observe the functional trend of the involved trig functions. These are shown in Figure 5.

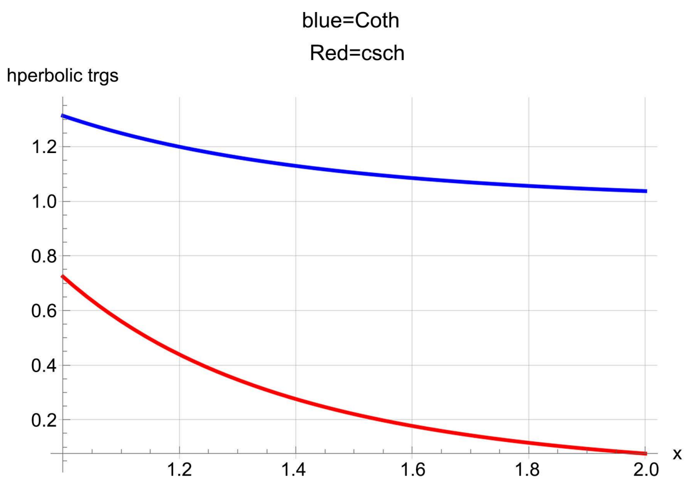

Figure 5. Display of the functional trend of the trig functions in Equation (7), coth and csch.

For the rest of the article, it is appropriate to make a symbolic notation change. This is because, in electrostatic problems, traditionally, the potentials are noted by V rather than φ. And because we are interested in intertwined circles along the x-axis separated by a distance d, the distance between the centers of the circles is,

$$d = x_{02} - x_{01}$$

Substituting for the coordinates of the centers of the circles given in the above text, we get,

$$d = a\left[\coth(V_1) - \coth(V_2)\right], \tag{8}$$

Since the capacitance is $c = q/(V_1 - V_2)$, noting the potentials are intertwined (8) by taking cosh of the potential difference, we separate them,

$$\cosh(V_1 - V_2) \equiv \cosh(V_1)\cosh(V_2) - \sinh(V_1)\sinh(V_2), \tag{9}$$

The right-most terms of (9) are substituted utilizing the right side of (7),

$$\sinh(V_1) = \frac{a}{R_1} \text{ and } \sinh(V_2) = \frac{a}{R_2}, \tag{10}$$

Reformatting (9),

$$\cosh(V_1 - V_2) = \sqrt{1+\sinh^2(V_1)}\sqrt{1+\sinh^2(V_2)} - \frac{a}{R_1}\frac{a}{R_2}, \tag{11}$$

On the other hand, (8) is written as,

$$a = d\frac{\sinh(V_1)\sinh(V_2)}{\cosh(V_1)\sinh(V_2)-\cosh(V_1)\sinh(V_2)}, \tag{12}$$

Manipulating (12) and crafting code yields,

$$f[R1\_, R2\_, a\_] = R1\sqrt{1+\left(\frac{a}{R1}\right)^2} - R2\sqrt{1+\left(\frac{a}{R2}\right)^2},$$

$$sola = Solve[f[R1, R2, a] == d, a],$$

$$\left\{\left\{a \to -\frac{\sqrt{d^4-2d^2R1^2+R1^4-2d^2R2^2-2R1^2R2^2+R2^4}}{2d}\right\}, \left\{a \to \frac{\sqrt{d^4-2d^2R1^2+R1^4-2d^2R2^2-2R1^2R2^2+R2^4}}{2d}\right\}\right\}$$

$$a = a\,/.\,sola$$

$$a[[2]]$$

$$\frac{\sqrt{d^4-2d^2R_1^2+R_1^4-2d^2R_2^2-2R_1^2R_2^2+R_2^4}}{2d}$$

Substituting the value of $a[[2]]$, (11) yields,

$$\cosh[V_1-V_2] = \frac{-\left(R1^2-R2^2\right)^2+d^2\left(R1^2+R2^2\right)}{2d^2R1R2},$$

On the other hand, $\left(R1^2-R2^2\right)^2 = d^4$, so that the last expression is,

$$\cosh[V_1-V_2] = \frac{-d^2+\left(R1^2+R2^2\right)}{2R1R2}, \tag{13}$$

And finally, the capacitance of the capacitor yields,

$$c = \frac{1}{\text{ArcCosh}\left(\frac{R_1^2+R_2^2-d^2}{2R_1R_2}\right)}, \tag{14}$$

If one applies the Gauss law, (14) will embody the auxiliary factors, resulting in capacitance per unit length of the capacitor shown in **Figure 1**.

$$\frac{c}{\ell} = \epsilon_0\left[\frac{2\pi}{\text{ArcCosh}\left(\frac{R_1^2+R_2^2-d^2}{2R_1R_2}\right)}\right], \tag{15}$$

2d. Numeric and Graphics

Applying (14) in practice is straightforward, but with caution. The separation distance between the centers of the cylinders is given by (8). Apply (10) for $a = 1$, it can be manipulated to yield,

$$d = R_1\sqrt{1+\frac{1}{R_1^2}} + R_2\sqrt{1+\frac{1}{R_2^2}}, \tag{16}$$

In other words, R_1, R_2, and d are related. By choosing a numeric value for R_1 and d, R_2 can be determined. By applying the CAS mentioned above [5], we seek its permissible value. For example, consider the radius of the large cylinder in Figure 1, which is colored blue, to be ten units, $R_1 = 10$. Separating the centers of the cylinders, $d = 1, \ldots, 9$ gives the allowed values of R_2, [6].

```
solR22=With[{R1=10.},Table[{d,NSolve[R2Sqrt[1+1/R2^2]==R1Sqrt[1+1/R1^2]-d,R2, Reals]},{d,1,9.,0.5}]]
```

```
{{1.,{{R2->8.99446}}},{1.5,{{R2->8.49119}}},{2.,{{R2->7.98752}}},{2.5,{{R2->7.48336}}},{3.,{{R2->6.97859}}},{3.5,{{R2->6.47309}}},{4.,{{R2->5.96666}}},{4.5,{{R2->5.45904}}},{5.,{{R2->4.94987}}},{5.5,{{R2->4.43862}}},{6.,{{R2->3.92447}}},{6.5,{{R2->3.40611}}},{7.,{{R2->2.88127}}},{7.5,{{R2->2.34561}}},{8.,{{R2->1.78941}}},{8.5,{{R2->1.18411}}},{9.,{{R2->0.319748}}}}
```

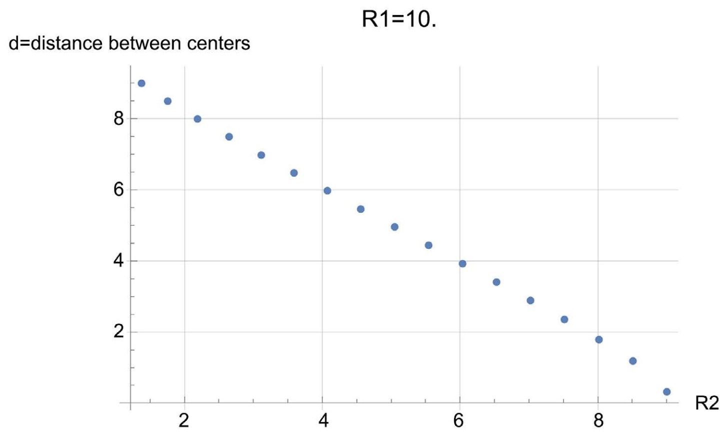

Figure 6. The allowed radii of the smaller cylinder in Figure 1 are the abasia.

In Figure 6, the vertical axis is the separation distance between the centers. The graph is plotted for the radius of the larger cylinder, $R_1 = 10$.

Here, we craft the *Mathematica* code aiming to graph the functional behavior of the capacitor (14). Noting also for a symmetric capacitor, *i.e.*, the one with a common cylindrical axis, the capacitance is $c \sim \frac{1}{\ln\left(\frac{\text{larger radius} = R_1}{\text{smaller radius} = R_2}\right)}$ [2] [3], its value is depicted with a red dot in the same graph, Figure 7.

```
listR2=R2/.Table[solR22[[n,2]],{n,1,Length[solR22]}]
```

```
{{8.99446},{8.49119},{7.98752},{7.48336},{6.97859},{6.47309},{5.96666},{5.45904},{4.94987},{4.43862},{3.92447},{3.40611},{2.88127},{2.34561},{1.78941},{1.18411},{0.319748}}
```

```
tabledata=With[{R1=10.},Table[{n,listR2[[n,1]],solR22[[n,1]],R1Sqrt[1+1/R1^2]-listR2[[n,1]]Sqrt[1+1/listR2[[n,1]]^2],(R1^2+listR2[[n,1]]^2-solR22[[n,1]]^2)/(2   *R1*listR2[[n,1]]),1/ArcCosh[(R1^2+listR2[[n,1]]^2-solR22[[n,1]]^2)/(2   *R1*listR2[[n,1]])]},{n,1,Length[solR22]}]]
```

```
{{1,8.99446,1.,1.,1.00006,89.9464},{2,8.49119,1.5,1.5,1.00016,56.6109},{3,7.98752,2.,2.,1.00031,39.9418},{4,7.48336,2.5,2.5,1.00056,29.939},{5,6.97859,3.,
```

3.,1.00092,23.2691},{6,6.47309,3.5,3.5,1.00146,18.5035},{7,5.96666,4.,4.,1.002 24,14.9278},{8,5.45904,4.5,4.5,1.00339,12.1449},{9,4.94987,5.,5.,1.00509,9.916 53},{10,4.43862,5.5,5.5,1.00765,8.09079},{11,3.92447,6.,6.,1.01162,6.5661},{12 ,3.40611,6.5,6.5,1.01805,5.27166},{13,2.88127,7.,7.,1.02909,4.15594},{14,2.345 61,7.5,7.5,1.04988,3.1793},{15,1.78941,8.,8.,1.09539,2.30744},{16,1.18411,8.5, 8.5,1.23097,1.49876},{17,0.319748,9.,9.,2.98708,0.568774}}

point=Graphics[{PointSize[0.05],Red,Point[{solR22[[16,1]],N[1/Log[listR 2[[17,1]]]]}]}];

Show[{ListPlot[Table[{tabledata[[n,4]],tabledata[[n,6]]},{n,1,Length[table data]}],AxesLabel->{"distance between centers","Capacitance"},GridLines-> Automatic,PlotRange->{{0,10},{-5,100}}],point}]

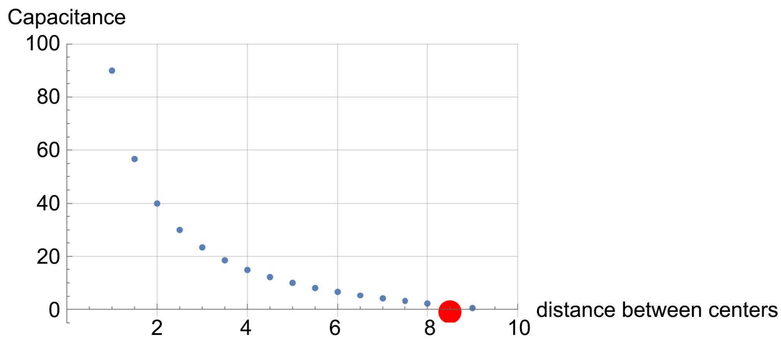

Figure 7. Display of the capacitance of a non-concentric capacitor, Equation (14), vs. the separation distance between the centers.

As explained, the red dot is the capacitance of a symmetric capacitor. The radii used to display their value are embedded in the code.

2d. Application

This section puts the analysis into action. Consider an RC-DC driven electric circuit. The voltage across the capacitor is [2] [3],

$$V(t) = V_0\left(1 - \mathrm{e}^{-\frac{t}{RC}}\right), \tag{17}$$

where V_0 is the voltage of the DC source, R and C are the values of the resistor and capacitor. For the sake of displaying the impact of a non-concentric capacitance, we compare the functional behavior of a symmetric vs. a non-symmetric capacitor with the same value of resistance, $R = 10$ units. The code accomplishes the goal:

voltageC[R_,t_]=1-e$^{-t/RC}$

plotGreen=Plot[voltageC[R,t]/.{C->tabledata[[15,6]],R->10.},{t,0,200}, PlotStyle->Red,PlotRange->{0,1}]

plotRed=Plot[voltageC[R,t]/.{C->(Log[10/tabledata[[15,2]]])$^{-1}$,R->10.}, {t,0,200},PlotStyle->Green,PlotRange->{0,1}]

In Figure 8, the voltage across the capacitors vs. time are depicted. The smaller capacitor in green with C of 1/Log[10/1.78] = 0.58 gets charged faster than the larger cap in red with the C of 1/ArcCosh[...] = 2.3.

{tabledata[[15,6]],(Log[10/tabledata[[15,2]]])$^{-1}$}={2.30744,0.581159}. The capacitance of the symmetric capacitor is ~173% smaller than that of the non-concentric one. And as expected, the larger capacitor, shown in red, reaches saturation voltage at a slower rate.

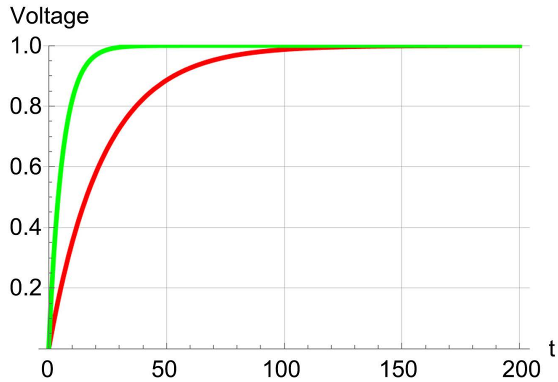

Figure 8. Charging voltage across the capacitors vs. time.

3. Conclusions and Remarks

This research-oriented study had one main goal. It is crafted as a complement to the numeric analysis of the previously published article [1]. It is shown that a non-concentric long cylindrical capacitor has a capacitance whose formulation depends on the radii of the parallel cylinders and the distance between the centers of the cylinders. Two innovative, non-traditional methods have been used to accomplish the goal. Applying *Mathematica* has been crucial for formulating the analytic aspect of the formulation. Additional information is presented both numerically and graphically. Codes are included, making the reproduction easy. The presented information has been used (applied) to analyze the impact of the asymmetry in a classic electric circuit.

The author is pursuing applying the shown methodology to a non-concentric spherical capacitor.

Individuals interested in *Mathematica* might find [5] [7] [8] resourceful.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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