

The *Njiki's* Fundamental Theorem-Definition on Fractions in the *Mathematical Set* \mathbb{Q} and by *Extension* in \mathbb{R} and \mathbb{C} , for the Purpose of Leading to the Construction of Some Algebraic Structures as Its Theoretical Applications and for the Practical Ones

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Abstract

The purpose of the research in the *NJIKI's* fundamental THEOREM-DEFINITION on fractions in the mathematical set \mathbb{Q} and by extension in \mathbb{R} and \mathbb{C} and in order to construct some algebraic structures is about the proved EXISTENCE and the DEFINITION by *NJIKI* of two INNOVATIVE, IMPORTANT and TEACHABLE operations of addition or additive operations, in \mathbb{Q} , marked \oplus and $+_{\alpha,\beta}$, and taken as VECTORIAL, TRIANGULAR, of THREE or PROPORTIONAL operations and in order to make THEM not be different from the RATIONAL ONE, +, but to bring much more and new information on fractions, and, by extension in \mathbb{R} and \mathbb{C} . And the very *NJIKI's* fundamental THEOREM-DEFINITION having many APPLICATIONS in the everyday life of the HUMAN BEINGS and without talking about computer sciences, henceforth being supplied with very interesting new ALGORITHMS. And as for the work done in the research, it will be waiting for its extension to be done after publication and along with the research results concerned.

Keywords

Operations of Addition, *Additive* Operations in \mathbb{Q} , Laws in \mathbb{Q} , THEOREM-DEFINITION, External Multiplication

1. Introduction

The present paper aims at stating the *NJIKI's* fundamental THEOREM-DEFINITION and *the ONE* being supported by two new operations of addition or additive operations in the *mathematical* set \mathbb{Q} , with along attached to *two* LEMMAS and subsidiarily *one* in two or *two* in one other nice *related NJIKI's* THEOREM and some EXERCISES, and not to forget the numerous and various algebraic structures to which it leads, in the Appendix B. And the research background so far therein lacking THEM all, and not to talk about the situation of the mathematical former researches as a whole and which had straight forward jumbled into \mathbb{R} and \mathbb{C} forgetting THEM back. And as for the list of the problems to be solved, it can be referred to in the end of PART 4 of the paper.

2. The Statement of the *NJIKI's* Fundamental THEOREM-DEFINITION

There *do* EXIST within the *mathematical* set \mathbb{Q} of fractions, besides the *traditional* addition $+$, for two *fractions' SUM* TOTAL calculation, and *mostly* for COUNTING and used by Serge Lang [1], Jean-Pierre Scoffier [2], Nicolas Bourbaki [3], Gilles Lachaud [4], Michel Demazure [5], among others, and the *modular* forms *somewhat* related to $+_{\alpha,1}$ or $+_{1,\beta}$, and usually referred to as the *5th* operation according to Gérard Eichler [6]-[9] who might have said it on day, and whereas indeed being between \oplus and $+_{\alpha,\beta}$ in *terms* or in *orders* of generalisation partially and totally when the *half*plane of Poincaré [10], gets involved into account, being put apart, *two* new other operations of addition or additive operations and *namely*: 1) the *natural* or *granulometric arithmetic* mean, \oplus , and 2) the *hybrid* or *composite arithmetic* mean, $+_{\alpha,\beta}$ with $(\alpha, \beta) \in \mathbb{Q}^{*2}$, and for two *fractions' SUM* MEANS calculation, and *mainly* for MEASURING, making them be taken as *NJIKI's* VECTORS or ALIKE, on one hand, and on the other hand as the *6th* and the *7th* operations due to *NJIKI* and as far as mathematics are concerned. They are DEFINED as follows:

$$1) \oplus : \mathbb{Q} \times \mathbb{Q} \rightarrow \mathbb{Q}$$

$$\left(\frac{b}{a}, \frac{d}{c}\right) \mapsto \oplus \left(\frac{b}{a}, \frac{d}{c}\right) = \frac{b}{a} \oplus \frac{d}{c} = \frac{b+d}{a+c} \tag{1}$$

$$2) +_{\alpha,\beta} : \mathbb{Q} \times \mathbb{Q} \rightarrow \mathbb{Q}$$

$$\left(\frac{b}{a}, \frac{d}{c}\right) \mapsto +_{\alpha,\beta} \left(\frac{b}{a}, \frac{d}{c}\right) = \frac{b}{a} +_{\alpha,\beta} \frac{d}{c} = \frac{\alpha b + \beta d}{\alpha a + \beta c} \tag{2}$$

Remarks: 1) \oplus and $+_{1,1}$ are identical and *that* is that $+_{\alpha,\beta}$ generalizes \oplus
 2) \oplus *makes* it *logically* clearer and *more* esthetic, *cases* generalization *including* $\frac{b}{a} = \frac{d}{c} = \frac{b+d}{a+c}$ ¹ among other *rational* numbers logics and *not* curiosities, and meaningful than the *addition* of DUNCE whose first user is FAREY [11], and through Lester R. Ford Sr [12]-[14], *mathematical* later works and not LOBACHEVSKI [15]-[17], and that according to the following proof.

¹See the Appendix.

3. Proof

1) Let

$$m_{a,c} \left(\frac{b}{a}, \frac{d}{c} \right) = \frac{b}{a} m_{a,c} \frac{d}{c} \tag{3}$$

or

$$m_{\frac{a}{a+c}, \frac{c}{a+c}} \left(\frac{b}{a}, \frac{d}{c} \right) = \frac{b}{a} m_{\frac{a}{a+c}, \frac{c}{a+c}} \frac{d}{c} \tag{4}$$

be the *natural* or the *granulometric* arithmetic mean of $\frac{b}{a}$ and $\frac{d}{c}$ and that is to say *that weighted* and *affected* with *weighing* coefficients a and c , or, *that other* weighted and affected with weighting coefficients $\frac{a}{a+c}$ and $\frac{c}{a+c}$ respectively. It is obviously or by *simple* calculations SHOWN from those above that:

$$\oplus = m_{a,c} \vee \oplus = m_{\frac{a}{a+c}, \frac{c}{a+c}} \tag{5}$$

And so then, the *EXISTENCE* of \oplus announced in the THEOREM and the DEFINITION following IT as well.

Now, as *how* to get or *come* about the *natural* or *granulometric weighting coefficients* a and c , or, $\frac{a}{a+c}$ and $\frac{c}{a+c}$, *one* and the *first* of the following others *NJIKI's THEOREMS enables* it. And the *which* can be taken as a LEMMA accordingly.

2) Let

$$m_{\alpha a, \beta c} \left(\frac{b}{a}, \frac{d}{c} \right) = \frac{b}{a} m_{\alpha a, \beta c} \frac{d}{c} \tag{6}$$

or

$$m_{\frac{\alpha a}{a+c}, \frac{\beta c}{a+c}} \left(\frac{b}{a}, \frac{d}{c} \right) = \frac{b}{a} m_{\frac{\alpha a}{a+c}, \frac{\beta c}{a+c}} \frac{d}{c} \tag{7}$$

be the hybrid or the composite arithmetic mean of $\frac{b}{a}$ and $\frac{d}{c}$ and that is to say that weighted and affected with weighing coefficients αa and βc , or, that other weighted and affected with weighting coefficients $\frac{\alpha a}{a+c}$ and $\frac{\beta c}{a+c}$ respectively. It is obviously or by *simple* calculations SHOWN from those above that:

$$+_{\alpha, \beta} = m_{\alpha a, \beta c} \vee +_{\alpha, \beta} = m_{\frac{\alpha a}{a+c}, \frac{\beta c}{a+c}} \tag{8}$$

And so then, the *EXISTENCE* of $+_{\alpha, \beta}$ announced in the THEOREM and the DEFINITION THAT follows IT as well.

Now, as *how* to get or *come* about the hybrid or composite weighting coefficients αa and βc , or, $\frac{\alpha a}{a+c}$ and $\frac{\beta c}{a+c}$, *one* and the *first* of the following others *NJIKI's THEOREMS enables* it. And the *which* can be taken as a LEMMA

accordingly and along with the observation preceding IT.

That above can so be taken as a very nice welcome back to the *Euclidian* geometry after almost 3 centuries from now back to N. I. LOBACHEVSKI, to be considered as the father of the *non-Euclidian* or *hyperbolic* geometries, and 20 centuries from LOBACHEVSKI to Euclid [18] [19]. And all things making *NJIKI* to become the *leader* or the *pioneer* of the neo Euclidian school in mathematics.

4. Some Other Theorems of Jean Claude NJIKI

4.1. The First Theorem of Jean Claude NJIKI or the Naturally or Granulometrically Weighted Arithmetic Mean Theorem of Jean Claude NJIKI, in $(\mathbb{Q}, +, \times)$, Taken as the LEMMA of the Above One

Given two fractions of the *field* $(\mathbb{Q}, +, \times)$, $\frac{b}{a}$ and $\frac{d}{c}$, and $(\alpha, \beta) \in \mathbb{Q}^{*2}$:

$$\frac{\alpha \frac{b}{a} + \beta \frac{d}{c}}{\alpha + \beta} = \frac{b+d}{a+c} \Leftrightarrow \begin{cases} \alpha = a \\ \beta = c \end{cases} \vee \begin{cases} \alpha = \frac{a}{a+c} \\ \beta = \frac{c}{a+c} \end{cases} \tag{9}$$

4.1.1. Observation

$\frac{\alpha \frac{b}{a} + \beta \frac{d}{c}}{\alpha + \beta}$ being the weighted *arithmetic* mean of $\frac{b}{a}$ and $\frac{d}{c}$ respectively and associated with the weighting *coefficients* α and β ; $\alpha = a \wedge \beta = c$ being their *natural* values and $\alpha = \frac{a}{a+c} \wedge \beta = \frac{c}{a+c}$ their *granulometric* ones, at the *right* side of the equivalence and from the *left* side equation of the very equivalence.

And by combining α and β with a and c , or, $\frac{a}{a+c}$ and $\frac{c}{a+c}$ respectively to have some *sorts* or *kinds* of hybrids or composites *weighting* coefficients αa and βc , or, $\frac{\alpha a}{a+c}$ and $\frac{\beta c}{a+c}$ to apply to the above α and β in their very places, it makes $\frac{\alpha b + \beta d}{\alpha a + \beta c}$ and in calculus whose $\lim_{\alpha \rightarrow \infty} \frac{\alpha b + \beta d}{\alpha a + \beta c} = \frac{b}{a}$ or $\lim_{\beta \rightarrow \infty} \frac{\alpha b + \beta d}{\alpha a + \beta c} = \frac{d}{c}$ and as some type of property of ratios or fractions going along with an application in DOCIMOMETRICS, and when it comes to talk about a *teaching* discipline with the *highest* weighting coefficient among two of them.

4.1.2. The Proof of the Theorem

The weighted *arithmetic* mean of two fractions of the *field* $(\mathbb{Q}, +, \times)$ $\frac{b}{a}$ and $\frac{d}{c}$ associated with the *weighting* coefficients α and β , with $(\alpha, \beta) \in \mathbb{Q}^{*2}$, is:

$\frac{\alpha \frac{b}{a} + \beta \frac{d}{c}}{\alpha + \beta}$. Let's set it equal to $\frac{b+d}{a+c}$ and let's resolve the two *linear* equation

systems S_1 and S_2 derived from that on or over $\mathbb{Q} \times \mathbb{Q} = \mathbb{Q}^2$ and in (α, β) . And let's say: $S_1 \begin{cases} \alpha \frac{b}{a} + \beta \frac{d}{c} = b+d \\ \alpha + \beta = a+c \end{cases}$ and $S_2 \begin{cases} \alpha \frac{b}{a} + \beta \frac{d}{c} = \frac{b+d}{a+c} \\ \alpha + \beta = 1 \end{cases}$. Their *common* de-

terminant is $\Delta_c = \begin{vmatrix} b & d \\ a & c \\ 1 & 1 \end{vmatrix} = \frac{b}{a} - \frac{d}{c} = \frac{bc-ad}{ac}$. It comes from the above for S_1 ,

that: $\alpha = \frac{\Delta_\alpha}{\Delta_c} = \frac{\begin{vmatrix} b+d & d \\ a+c & 1 \end{vmatrix}}{\Delta_c} = a$, and $\beta = \frac{\Delta_\beta}{\Delta_c} = \frac{\begin{vmatrix} b & b+d \\ 1 & a+c \end{vmatrix}}{\Delta_c} = c$; and for S_2 , that:

$\alpha = \frac{\Delta_\alpha}{\Delta_c} = \frac{\begin{vmatrix} b+d & d \\ a+c & c \\ 1 & 1 \end{vmatrix}}{\Delta_c} = \frac{a}{a+c}$, and $\beta = \frac{\Delta_\beta}{\Delta_c} = \frac{\begin{vmatrix} b & b+d \\ a & a+c \\ 1 & 1 \end{vmatrix}}{\Delta_c} = \frac{c}{a+c}$. And let's say to

conclude that $S_n = \{(\alpha, \beta) = (a, c)\}$ for the set of the *natural* solutions and

$S_g = \left\{(\alpha, \beta) = \left(\frac{a}{a+c}, \frac{c}{a+c}\right)\right\}$ for that of *granulometric* solutions.

4.2. Other, Properly Said, Theorems of Jean Claude NJIKI within $(\mathbb{Q}, +, \times)$

4.2.1. The Second Theorem of Jean Claude NJIKI or the SUM TOTAL of Two Ratios or Fractions Theorem of Jean Claude NJIKI, in $(\mathbb{Q}, +, \times)$

Given two *ratios* or fractions, $\frac{b}{a}$ and $\frac{d}{c}$ of $(\mathbb{Q}, +, \times)$, and $(\alpha, \beta) \in \mathbb{Q}^{*2}$:

$$\frac{\alpha a + \beta d}{\alpha a + \beta c} = \frac{b}{a} + \frac{d}{c} \Leftrightarrow bc - ad \neq 0 \tag{10}$$

Proof

Traditionally or classically, basically:

Given five *numbers* a, b, c, d and k of \mathbb{Z} and with $a \neq 0 \wedge c \neq 0$:

$$\frac{b}{a} + \frac{d}{c} = \frac{bc+ad}{ac}$$

Now let's name A and B the two *logical* propositions to the *left* side and at the *right* side of the *logical* equivalence.

Let's show that $B \Rightarrow A$. That goes with saying; and because

$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} b & d \\ a & c \end{pmatrix}^{-1} \begin{pmatrix} bc+ad \\ ac \end{pmatrix}$ and *meaning* the existence of $(\alpha, \beta) \in \mathbb{Q}^{*2}$. And

which is the *aim* to achieve.

Now let's show that $A \Rightarrow B$, and which amounts to, and *by* contraposition, to show that $\neg B \Rightarrow \neg A$, $\neg B$ signifies that $bc - ad = 0$ and that is to say that

$\frac{b}{a} = \frac{d}{c}$ or, in others terms, that $b = ka \wedge d = kc$, by setting $\frac{b}{a} = \frac{d}{c} = k$. By

transferring these values found in $\neg B$ in $\neg A$ meaning that

$\frac{\alpha b + \beta d}{\alpha a + \beta c} \neq \frac{b}{a} + \frac{a}{c} = \frac{bc + ad}{ac}$ we obtain effectively successively k et $2k$ and then the result and because $k \neq 2k$.

4.2.2. The Third Theorem of Jean Claude NJIKI or the Simple Arithmetic Mean of Two Fractions Theorem of Jean Claude NJIKI, in $(\mathbb{Q}, +, \times)$

Given two ratios or fractions, $\frac{b}{a}$ and $\frac{d}{c}$ of $(\mathbb{Q}, +, \times)$, and $(\alpha, \beta) \in \mathbb{Q}^{*2}$:

$$\frac{\alpha b + \beta d}{\alpha a + \beta c} = \frac{1}{2} \left(\frac{b}{a} + \frac{d}{c} \right) \Leftrightarrow bc - ad \neq 0 \vee bc - ad = 0 \tag{11}$$

And for the proof, the right side to the left side implication part of it, the equivalence, is alright in the 1st third of the three cases (when $bc - ad \neq 0$), and that from the proof of the previous second theorem and by disjunction of cases making $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} bd \\ ac \end{pmatrix}^{-1} \begin{pmatrix} bc + ad \\ 2ac \end{pmatrix}$ and for the 2nd third (when $bc - ad = 0$ and meaning that $\frac{b}{a} = \frac{d}{c}$), the result $k = k$ is achieved by setting $\frac{b}{a} = \frac{d}{c} = k$ and which gives $b = ka \wedge d = kc$, and the conjunction of both $(bc - ad \neq 0 \wedge bc - ad = 0)$ constituting the 3rd third and being always FALSE leads to the expected implication. For the left side to the right side implication part of the equivalence now and meaning the implication of the negation of the right side to the negation of the left side, the resulted implication is straightly obtained and because the negation of the right side $(bc - ad = 0 \wedge bc - ad \neq 0)$ is always FALSE.

Remark: the second and the third theorems of Jean Claude NJIKI can be put together and as follows: the algebraic form $\frac{\alpha b + \beta d}{\alpha a + \beta c}$ can give $\frac{b}{a} + \frac{d}{c}$ under the condition of $bc - ad \neq 0$ or $\frac{1}{2} \left(\frac{b}{a} + \frac{d}{c} \right)$ in any case.

A vector VERSION, in the FORM, of the NJIKI's fundamental THEOREM-DEFINITION does exist. And it's through the DISCOVERY by NJIKI of the \mathbb{Z} -module $(\mathbb{Q}_{jcn}, +, \bullet)$ with $\mathbb{Q}_{jcn} = \mathbb{Z} \times \mathbb{Z}$ including \mathbb{Q} ($\mathbb{D} \subset \mathbb{Q} \subset \mathbb{Q}_{jcn}$) and DEFINED as follows:

$$\mathbb{Q}_{jcn} = \left\{ [(a, b), (c, d)] \in (\mathbb{Z} \times \mathbb{Z})^2 / \forall (\alpha, \beta) \in \mathbb{Z}^2, \alpha \begin{pmatrix} a \\ b \end{pmatrix} + \beta \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} \alpha a + \beta c \\ \alpha b + \beta d \end{pmatrix} = \begin{pmatrix} ac \\ bd \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right\}, \text{ tangentially.} \tag{12}$$

or

$$\mathbb{Q}_{jcn} = \left\{ [(a, b), (c, d)] \in (\mathbb{Z} \times \mathbb{Z})^2 / \forall (\alpha, \beta) \in \mathbb{Z}^2, \alpha \begin{pmatrix} b \\ a \end{pmatrix} + \beta \begin{pmatrix} d \\ c \end{pmatrix} = \begin{pmatrix} \alpha b + \beta d \\ \alpha a + \beta c \end{pmatrix} = \begin{pmatrix} bd \\ ac \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right\}, \text{ co-tangentially.} \tag{13}$$

Now, with the field $(\mathbb{Q}, +, \times)$, in the place of the ring $(\mathbb{Z}, +, \times)$, the above \mathbb{Z} -module becomes the *NJIKI's* \mathbb{Q} -vector plane $(\mathbb{Q}_{jcn}, +, \bullet)$.

And all things making FRACTIONS not only to be viewed as RATIONAL and as THEY were so far till now but somewhat also as VECTORIAL.

And the game between the numerator b and the denominator a of a given fraction $\frac{b}{a} \in \mathbb{Q}$, and, the 1^{st} and the 2^{nd} coordinates a and b of its corresponding vector $\begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{Q}_{jcn}$ or $\begin{pmatrix} b \\ a \end{pmatrix}$ vice versa depending upon whether one works tangentially or cotangentially.

The mayor two levels of discussion between \mathbb{Q} and \mathbb{Q}_{jcn} being at those: 1) of the *equivalence* classes or *classes* of equivalence of \mathbb{Q} , formalised by $\frac{kb}{ka} = \frac{b}{a}$, and, their *corresponding vector* collinearity or homothety relation in \mathbb{Q}_{jcn} , defined as $k \begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} kb \\ ka \end{pmatrix}$, and 2) the euclidian division in \mathbb{Q} , translated by the *affine line equation* $b = aq + r$ with $0 \leq r < a$, and, its *matching vector line equation* $B = Aq$ due to *NJIKI* and *associated* to his divisibility and obtained by the *vector* translation or the *variables* change defined as follows $\begin{cases} B = b - r \\ A = a \end{cases}$,

With $(a, b, q, r, A, B) \in \mathbb{Z}^6$.

And the *BEST* of IT, and that is to say of the *NJIKI's* fundamental THEOREM-DEFINITION, BEING within *ITS VARIOUS APPLICATIONS* and in the *everyday human life ACTIVITIES including* DOCIMOMETRICS, PROBABLIMETRICS and ELECTION-METRICS due to *NJIKI* and defined as *MEASURE* in DOCIMOLOGY, PROBABILITY and ELECTIONS, in politics among others, and respectively, to name but THREE of THEM, and many OTHERS, and still NEOLOGISMS as well, and form *NJIKI* or not, involving SHARING or DIVISION situations or cases, being yet to come, in teaching and elsewhere, later on.

5. Exercise of Application in Mathematics Themselves or Per Se and for Gathering Much More Knowledge around the *NJIKI's* Fundamental THEOREM-DEFINITION

Show the following:

(A) Given $(a, b, c, d) \in \mathbb{Z}^4 \wedge (a \neq 0 \wedge c \neq 0) \wedge (\alpha, \beta) \in \mathbb{Z}^{*2}$,

$$\left(\frac{b}{a} \oplus \frac{d}{c}\right) + \frac{c}{a+c} \frac{b}{a} + \frac{a}{a+c} \frac{d}{c} = \frac{bc+ad}{ac} = \frac{b}{a} + \frac{d}{c}$$

$$\left(\frac{b}{a} +_{\alpha, \beta} \frac{d}{c}\right) + \frac{\beta c}{\alpha a + \beta c} \frac{b}{a} + \frac{\alpha a}{\alpha a + \beta c} \frac{d}{c} = \frac{b}{a} + \frac{d}{c}$$

(B) $\forall \left(\frac{b}{a}, \frac{d}{c}\right) \in (\mathbb{Q}, +, \times)$

$$(1) \quad \frac{\alpha b + \beta d}{\alpha a + \beta c} = \frac{b}{a} + \frac{d}{c} \quad \longmapsto \quad \alpha a^2 d + \beta b c^2 = 0$$

$$\begin{aligned}
 (2) \quad & \frac{\alpha b + \beta d}{\alpha a + \beta c} = \frac{1}{2} \left(\frac{b}{a} + \frac{d}{c} \right) && \vdash (bc - ad)(\alpha a - \beta c) = 0 \\
 (2)' \quad & \frac{b + d}{a + c} = \frac{1}{2} \left(\frac{b}{a} + \frac{d}{c} \right) && \vdash (bc - ad)(a - c) = 0 \\
 (3) \quad & \frac{b + d}{a + c} = \frac{1}{\alpha + \beta} \left(\alpha \frac{b}{a} + \beta \frac{d}{c} \right) && \vdash (bc - ad)(\beta a - \alpha c) = 0 \\
 (3)' \quad & \frac{\alpha b + \beta d}{\alpha a + \beta c} = \frac{1}{\alpha + \beta} \left(\alpha \frac{b}{a} + \beta \frac{d}{c} \right) && \vdash (bc - ad)(\alpha \beta a - \alpha \beta c) = 0 \\
 & && \vdash \alpha \beta (bc - ad)(a - c) = 0 \\
 (4) \quad & \frac{1}{2} \left(\frac{b}{a} + \frac{d}{c} \right) = \frac{1}{\alpha + \beta} \left(\alpha \frac{b}{a} + \beta \frac{d}{c} \right) && \vdash (bc - ad)(\alpha ac - \beta ac) \\
 & && \vdash ac(bc - ad)(\alpha - \beta) = 0 \\
 (4)' \quad & \frac{\alpha b + \beta d}{\alpha a + \beta c} = \frac{b + d}{a + c} && \vdash (\alpha - \beta)(bc - ad) \\
 & && = 0 \vee (\beta - \alpha)(bc - ad) = 0 \\
 (0) \quad & \frac{\alpha b + \beta d}{\alpha a + \beta c} = \frac{b}{a} + \frac{d}{c} && \vdash bc - ad \neq 0 \\
 (3) \quad & \frac{b + d}{a + c} = \frac{1}{\alpha + \beta} \left(\alpha \frac{b}{a} + \beta \frac{d}{c} \right) && \vdash \alpha = \frac{a}{a + c} \wedge \beta = \frac{c}{a + c} \\
 (\text{bis}) \quad & && \\
 (4)' \quad & \frac{\alpha b + \beta d}{\alpha a + \beta c} = \frac{b + d}{a + c} && \vdash \alpha = \beta = 1 \\
 (\text{bis}) \quad & && \\
 (C) \quad & +_{\alpha, \beta} \left(\frac{b}{a}, \frac{d}{c} \right) = m_{\frac{\alpha a}{\alpha a + \beta c}, \frac{\beta c}{\alpha a + \beta c}} \left(\frac{b}{a}, \frac{d}{c} \right)
 \end{aligned}$$

And A, B and C being referred to as the *NJIKI's mathematical IDENTITIES*, the *NJIKI's logical EQUIVALENCES* and the *NJIKI's \mathcal{F}^d value of $+_{\alpha, \beta}$* named the “*weighted-granulometric*” one, $+_{\alpha, \beta} = m_{\frac{\alpha a}{\alpha a + \beta c}, \frac{\beta c}{\alpha a + \beta c}}$, and besides: 1)

$+_{\alpha, \beta} = m_{\alpha a, \beta c}$ and, 2) $+_{\alpha, \beta} = m_{\frac{\alpha a}{a + c}, \frac{\beta c}{a + c}}$, respectively. And definitely:

$$+_{\alpha, \beta} \in \left\{ m_{\alpha a, \beta c}, m_{\frac{\alpha a}{a + c}, \frac{\beta c}{a + c}}, m_{\frac{\alpha a}{\alpha a + \beta c}, \frac{\beta c}{\alpha a + \beta c}}, \dots \right\}.$$

6. Discussion or Conclusion

It can be done in two parts and as follows:

1) A mathematical study of the relation of equality, marked $=_{1,1}$, or, $=_{\alpha, \beta}$, and knowing that $=_{1,1} = =_{\alpha, \beta}$, within the \mathbb{Q} equivalence classes or classes of equivalence $\frac{\dot{b}}{a}$ or $\frac{\dot{d}}{c}$.

The relation of equality, marked $=_{1,1}$, or, $=_{\alpha, \beta}$, within the \mathbb{Q} equivalence classes or classes of equivalence $\frac{\dot{b}}{a}$ or $\frac{\dot{d}}{c}$, is an ADDITIVE internal composi-

tion law and as an application defined as follows:

$$=_{1,1}, \text{ or, } =_{\alpha,\beta} : \frac{\dot{b}}{a} \times \frac{\dot{b}}{a} \rightarrow \frac{\dot{b}}{a}$$

$$\left(\frac{b}{a}, \frac{d}{c}\right) \mapsto =_{\alpha,\beta} \left(\frac{b}{a}, \frac{d}{c}\right) = \frac{b}{a} =_{\alpha,\beta} \frac{d}{c} = \frac{b+d}{a+c} = \frac{\alpha a + \beta d}{\alpha a + \beta c}$$

or

$$=_{1,1}, \text{ or, } =_{\alpha,\beta} : \frac{\dot{d}}{c} \times \frac{\dot{d}}{c} \rightarrow \frac{\dot{d}}{c}$$

$$\left(\frac{b}{a}, \frac{d}{c}\right) \mapsto =_{1,1} \left(\frac{b}{a}, \frac{d}{c}\right) = \frac{b}{a} =_{1,1} \frac{d}{c} = \frac{b+d}{a+c} = \frac{\alpha a + \beta d}{\alpha a + \beta c}$$

From the geometry view point, the relation of equality $\frac{b}{a} = \frac{d}{c}$, algebraically defining an ADDITIVE operation or an ADDITIVE internal composition law and as an application $\left(\frac{b}{a}, \frac{d}{c}\right) \mapsto \frac{b}{a} = \frac{d}{c} = \frac{b+d}{a+c}$, of $\frac{\dot{b}}{a} \times \frac{\dot{b}}{a} \rightarrow \frac{\dot{b}}{a}$ or $\frac{\dot{d}}{c} \times \frac{\dot{d}}{c} \rightarrow \frac{\dot{d}}{c}$ in \mathbb{Q} , does reflect the theorem of THALES studied by Rudolf Bkouche [20], and as it can be found in the “Algèbre et geometrie” of Michel Demazure [5], and as well as the *NJIKI*'s suites $s_k = \frac{kb}{ka}$ or $s_h = \frac{hd}{hc}$, in \mathbb{Q} , do reflect the extension of the very theorem, in the real affine plane \mathbb{R}^2 .

In analysis or calculus, the equality or proportion $\frac{b}{a} = \frac{d}{c}$, algebraically also meaning that $\frac{b}{a} = \frac{d}{c} = \frac{b-d}{a-c} = \frac{d-b}{c-a}$, can be linked to the theorem of ROLLS [21]-[25], found into Hervé Queffelec [26], and Jacques Baranger's [27], books.

And that so from the *NJIKI*'s values a and c of a certain variable x , and, those b and d respectively of their images by the function y as follows and ACCURATELY:

$$\frac{\Delta y}{\Delta c} = \frac{b-d}{a-c} = \frac{d-b}{c-a} = \frac{b}{a} = \frac{d}{c}$$

Verification

By the means of the following two parts, a) differential and b) integral, of the infinitesimal calculation and therein for the purpose of some historical revival and since the infinitesimal calculation is at the origin of the modern analysis or calculus:

- a) Given that $y = \frac{b}{a}x$ or $y = \frac{d}{c}x$, as the vector line version of the matter, or, $y = \frac{b}{a}x + e$ or $y = \frac{d}{c}x + f$, as its associated affine straight line:

$$y' = \lim_{x \rightarrow x_0} \frac{y - y_0}{x - x_0} = \frac{b}{a} = \frac{d}{c}$$

- b)

$$y = \int_0^x \frac{b}{a} du = \left[\frac{b}{a} u \right]_0^x = \frac{b}{a} x + e$$

$$y = \int_0^x \frac{d}{c} dv = \left[\frac{d}{c} v \right]_0^x = \frac{d}{c} x + f$$

And both (a) and (b), to conclude, dealing with theories of measure respectively associated to fractions and related to *NJIKI's* suites in \mathbb{Q} , and to integral in \mathbb{R}^2 .

In its meaning, the algebraic form $\frac{\alpha a + \beta d}{\alpha a + \beta c}$, in \mathbb{Q} , represents a certain weighted arithmetic MEAN VALUE of $\frac{b}{a}$ and $\frac{d}{c}$ affected with suitable weighting coefficients and whether $\frac{b}{a} \neq \frac{d}{c}$ or $\frac{b}{a} = \frac{d}{c}$ meaning in any case. And it does go the same way with $\frac{a+d}{a+c}$, in \mathbb{Q} , and particularly then in $\frac{b}{a}$ or $\frac{d}{c}$ of \mathbb{Q} , for the above both $\frac{\alpha a + \beta d}{\alpha a + \beta c}$ and $\frac{b+d}{a+c}$. More of facts, the algebraic form $\frac{b+d}{a+c}$ and from $\frac{b}{a}$ and $\frac{d}{c}$, also found in the hyperbolic geometry, developed in the 18th century by LOBACHEVSKI [3], and then by Poincare, can now be given much more MEANING and THAT from the euclidian geometry and more generally in the affine geometry.

Besides the above, it is obvious that the background and the results of the Article here are linked, for its title “the *NJIKI's* fundamental THEOREM-DEFINITION on fractions in the mathematical set \mathbb{Q} and by extension in \mathbb{R} and \mathbb{C} ” does speak itself and by showing: a) the LACK of some THEORETICAL KNOWLEDGE about \mathbb{Q} and by extension in \mathbb{R} and \mathbb{C} , as far as the said background is concerned, and, b) the coverage of that GAP, and associated with the practical applications in so many areas of the utilisation of fractions including the mathematics themselves and others namely DOCIMOMETRICS, PROBABLIMETRICS and ELECTION-METRICS, for example, as for the results. And not to forget as the result of results, the very wide transmission, around or over the WORLD, of the ABOVE BOTH, the improved theoretical knowledge about fractions carried up “by the *NJIKI's* fundamental THEOREM-DEFINITION on fractions...” and its various practical applications, and the very wide transmission, around or over the WORLD, of the ABOVE BOTH ENABLED, through publication, by the American Journal of Computational Mathematics (AJCM) sponsored by the Scientific Research Publishing (SCIRP)!!! Warm congratulations and everlasting life to the AJCM/SCIRP then and for making “The *NJIKI's* fundamental THEOREM-DEFINITION on fractions in the mathematical set \mathbb{Q} and by extension in \mathbb{R} and \mathbb{C} ” worldwide known and shared and for the wellbeing all-round of the mankind.

2) A very short *mathematical* study between the *relation* of equality, marked $=_{1,1}$, or, $=_{\alpha,\beta}$, and knowing that $=_{1,1} = =_{\alpha,\beta}$, within the \mathbb{Q} *equivalence* classes or classes of equivalence $\frac{b}{a}$ or $\frac{d}{c}$, the additive operations or the operations of

ADDITION $+_{1,1}$ and $+_{\alpha,\beta}$, in \mathbb{Q} , and that so, from the generalization view Point: $+_{1,1}$, in \mathbb{Q} , generalizes $=_{1,1}$, or, $=_{\alpha,\beta}$, in $\frac{\dot{b}}{a}$ or $\frac{\dot{d}}{c}$ of \mathbb{Q} , and, $+_{\alpha,\beta}$, in \mathbb{Q} , generalizes $+_{1,1}$, in \mathbb{Q} , and then $=_{1,1}$, or, $=_{\alpha,\beta}$, in $\frac{\dot{b}}{a}$ or $\frac{\dot{d}}{c}$ of \mathbb{Q} , by the *property* of transitivity of the relation of generalisation, in \mathbb{Q} .

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix

(A) An INNOVATIVE mathematical study of the *logical* propositional expression $\frac{b}{a} = \frac{d}{c} = \frac{b+d}{a+c}$, in \mathbb{Q} :

Classically proved by setting $\frac{b}{a} = \frac{d}{c} = k$, and among *other* methods, leading successively to $k = \frac{b+d}{a+c}$ and then to $\frac{b}{a} = \frac{d}{c} = \frac{b+d}{a+c}$, the result, the above logical propositional expression can be viewed as the beginning of a *NJIKI's digital ACCURATE* suite $s_k = \left(\frac{b}{a}\right)_k = \frac{kb}{ka}$, with $\frac{b}{a}$ being irreducible and

$k \in \mathbb{Z}^*$ while $\lim_{k \rightarrow \infty} s_k = \frac{b}{a}$ and meaning that s_k is constant or stationary, or, $s_h = \left(\frac{d}{c}\right)_h = \frac{hd}{hc}$, with $\frac{d}{c}$ being irreducible and $h \in \mathbb{Z}^*$ while $\lim_{h \rightarrow \infty} s_h = \frac{d}{c}$ and

meaning that s_h is constant or stationary, and the very *NJIKI's digital ACCURATE* suite, s_k or s_h , DEFINING each an *equivalence* class or *class* of equivalence of \mathbb{Q} . On *one* way and in CALCULUS that is rigorously and accurately far better than the *suites* of Cauchy [28], with reals, complexes or points of a *metric* space and more generally of an *uniform* space. And on the *other* way and in GENERAL ALGEBRA, in the *double* equality $\frac{b}{a} = \frac{d}{c} = \frac{b+d}{a+c}$, the *second* one, =, is proper and the *first* one, =, can be considered as an *internal* composition law *marked* * and of an *ADDITIVE* nature, *logically* and *aesthetically* far better than the *ADDITION* of DUNCE, and so as to make it be $\frac{b}{a} * \frac{d}{c} = \frac{b+d}{a+c}$. And that from

$\frac{\dot{b}}{a} \times \frac{\dot{d}}{c}$ to $\frac{\dot{d}}{a}$, or, from $\frac{\dot{d}}{c} \times \frac{\dot{d}}{c}$ to $\frac{\dot{d}}{c}$, and not from $\mathbb{Q} \times \mathbb{Q}$ to \mathbb{Q} where the *function* * is not even an application and because of the separation of its *equivalence* classes or *classes* of equivalence. And now, to *formally* summarize the above:

$$* : \frac{\dot{b}}{a} \times \frac{\dot{b}}{a} \rightarrow \frac{\dot{b}}{a}$$

$$\left(\frac{b}{a}, \frac{d}{c}\right) \mapsto * \left(\frac{b}{a}, \frac{d}{c}\right) = \frac{b}{a} * \frac{d}{c} = \frac{b+d}{a+c}$$

or

$$* : \frac{\dot{d}}{c} \times \frac{\dot{d}}{c} \rightarrow \frac{\dot{d}}{c}$$

$$\left(\frac{b}{a}, \frac{d}{c}\right) \mapsto * \left(\frac{b}{a}, \frac{d}{c}\right) = \frac{b}{a} * \frac{d}{c} = \frac{b+d}{a+c}$$

And all things *showing* or what *can* prove that, or, which *can be* shown or *be* proved from that:

* = = \vee = = *, and where = is the *1st* one equality of the above double equali-

ty $\frac{b}{a} = \frac{d}{c} = \frac{b+d}{a+c}$ and being a *particular* case of $+_{1,1}$, and so so, with a very *good* tautology ($= \vee =$) behind, besides.

And for the *recap* of the major items: the *operation* of ADDITION $+_{\alpha,\beta}$, in \mathbb{Q} , generalizes THOSE of the *modular* forms related to $+_{\alpha,1}$ or $+_{1,\beta}$, in \mathbb{Q} , which THEMSELVES generalize THAT \oplus or $+_{1,1}$, in \mathbb{Q} , which ITSELF generalizes THAT other $=$ or $*$, in $\frac{\dot{b}}{a}$ or $\frac{\dot{d}}{c}$, belonging to \mathbb{Q} and as one of its *infinite equivalence* classes or *classes* of equivalence corresponding to the ACCURATE *suites* of NJIKI s_k or s_h and among many numerous *infinite* others.

EXERCISE of APPLICATION in mathematics themselves or per se.

On the above pattern, make mathematically, in \mathbb{Q} and from the analysis or *calculus view* point, and, in $\frac{\dot{b}}{a}$ or $\frac{\dot{d}}{c}$ and from the *algebra* one, an

INNOVATIVE STUDY of the *logical* proposition $\frac{b}{a} = \frac{d}{c} = \frac{\alpha b + \beta d}{\alpha a + \beta c}$;

With $= = *_{\alpha,\beta} \vee *_{\alpha,\beta} = =$

Notice that in $\frac{\dot{b}}{a}$ or $\frac{\dot{d}}{c}$, $*_{\alpha,\beta} = *_{1,1} = *$, meaning that there is no need of talking about any *generalization* between $*_{\alpha,\beta}$ and $*_{1,1}$, in $\frac{\dot{b}}{a}$ or $\frac{\dot{d}}{c}$, and, as it is the case between $+_{\alpha,\beta}$ and $+_{1,1}$, in \mathbb{Q} .

And what about the *rational* number $\frac{\alpha b + \beta d}{\alpha a + \beta c} \in \mathbb{Q}$ and the *vector*

$$\begin{pmatrix} \alpha a + \beta c \\ \alpha b + \beta d \end{pmatrix} \in \mathbb{Q}_{jcn} ?$$

A *partial* mathematical study of the *rational* number $\frac{\alpha b + \beta d}{\alpha a + \beta c} \in \mathbb{Q}$ and the *vector* $\begin{pmatrix} \alpha a + \beta c \\ \alpha b + \beta d \end{pmatrix} \in \mathbb{Q}_{jcn}$ and \mathbb{Q}_{jcn} being considered as that of the *Njiki's* Z -module $(\mathbb{Q}_{jcn}, +, \cdot)$.

The *rational* number $\frac{\alpha b + \beta d}{\alpha a + \beta c} \in \mathbb{Q}$ is of the same *class* of equivalence or *equivalence* class than $\frac{b}{a}$ and $\frac{d}{c}$ in case that $\frac{b}{a} = \frac{d}{c}$ and *what* makes it be

$\frac{b}{a} = \frac{d}{c} = \frac{\alpha b + \beta d}{\alpha a + \beta c}$ and *which* is a *particular* case of the *general* case

$\frac{b}{a} +_{\alpha,\beta} \frac{d}{c} = \frac{\alpha b + \beta d}{\alpha a + \beta c}$ and that is to say that even when $\frac{b}{a} \neq \frac{d}{c}$.

Now, in *all* cases or in *any* case, meaning whether $\frac{b}{a} = \frac{d}{c}$ or $\frac{b}{a} \neq \frac{d}{c}$, the *rational* number $\frac{\alpha b + \beta d}{\alpha a + \beta c} \in \mathbb{Q}$ and the *vector* $\begin{pmatrix} \alpha a + \beta c \\ \alpha b + \beta d \end{pmatrix}$ do describe or characterize the *same* *vector* line and its associated *affine* straight line, and as their

slope, in the *field* $(\mathbb{Q}, +, \times)$, and *vector director*, in the *NJIKI's* \mathbb{Z} -module $(\mathbb{Q}_{jcn}, +, \cdot)$, respectively.

(B) About the *existence* of an *external multiplication operating* over the set \mathbb{Q}

There does *exist* an *external multiplication operating* over the set \mathbb{Q} , with scalars in \mathbb{Z} or \mathbb{Q} , *marked* \bullet , and defined as follows:

$$\bullet : \mathbb{Z} \times \mathbb{Q} \rightarrow \mathbb{Q}$$

$$\left(\lambda, \frac{b}{a}\right) \mapsto \bullet \left(\lambda, \frac{b}{a}\right) = \lambda \bullet \frac{b}{a} = \frac{\lambda b}{\lambda a} = \frac{b}{a}$$

or

$$\bullet : \mathbb{Q} \times \mathbb{Q} \rightarrow \mathbb{Q}$$

$$\left(\mu, \frac{d}{c}\right) \mapsto \bullet \left(\mu, \frac{d}{c}\right) = \mu \bullet \frac{d}{c} = \frac{\mu d}{\mu c} = \frac{d}{c}$$

OBSERVATION

The *external multiplication* \bullet can be taken as the *identity* of \mathbb{Q} , $id_{\mathbb{Q}}$, or as a *vector homothety* of \mathbb{Q} of *ratio* 1, h_1 . It also besides permits some very good definitions of both: 1) the \mathbb{Q} equivalence classes or classes of equivalence, $\frac{\dot{b}}{a}$,

and 2) the *NJIKI's digital ACCURANTE* suite, $\left(\frac{b}{a}\right)_\lambda$, and as follows:

$$1) \frac{\dot{b}}{a} = \left\{ \frac{d}{c} \in \mathbb{Q} / \frac{d}{c} = \lambda \cdot \frac{b}{a} \wedge \lambda \in \mathbb{Z}^* \right\}$$

$$2) \left(\frac{b}{a}\right)_\lambda = \lambda \cdot \frac{b}{a} \text{ with } \lambda \in \mathbb{Z}^*$$

It can easily be proved that:

1) The *external multiplication* \bullet is linked to $+_{1,1}$ as follows:

$$\lambda \bullet \frac{b}{a} = \underbrace{\frac{b}{a} +_{1,1} \dots +_{1,1} \frac{b}{a}}_{\lambda \text{ times } \frac{b}{a}} \text{ (by the definition of an operation of multiplication and$$

by decomposition or development, in one way);

$$= \frac{b + \dots + b}{a + \dots + a} \text{ (by the definition of } +_{1,1} \text{ and its obvious associativity's}$$

property, $\lambda - 2$ times);

$$= \frac{\lambda b}{\lambda a} \text{ (by the definition of an operation of multiplication and by factorisation,$$

on another way);

$$= \frac{b}{a} \text{ (by the } \mathbb{Q} \text{ equivalence classes or classes of equivalence's property or by$$

the fractions simplification as well).

Or

$$\mu \bullet \frac{d}{c} = \underbrace{\frac{d}{c} +_{1,1} \dots +_{1,1} \frac{d}{c}}_{\mu \text{ times } \frac{d}{c}} = \frac{d + \dots + d}{c + \dots + c} = \frac{\mu d}{\mu c} = \frac{d}{c},$$

and within the \mathbb{Q} equivalence classes or *classes* of equivalence the *additive* operation $=_{1,1}$ can also come in good handy in the place of $+_{1,1}$, assuming λ or μ being (*natural* wholes) of \mathbb{N} , to make the *enumeration* constituting the *heart* of the proof easier, and by *extension* of \mathbb{Z} and \mathbb{Q} , by some *game* of signs, $-$ and $+$, between \mathbb{Z} and \mathbb{N} , and, *that* of proportionality, by *vector* translation or *change* of reference bringing the *Euclidian* division in to the *NJIKI's* divisibility, between \mathbb{Q} and (\mathbb{Z} or \mathbb{N}).

2) The *algebraic* structure $\left(\mathbb{Q} \cup \left\{\frac{b}{0} / b \in \mathbb{Z}\right\}, +_{1,1}, \bullet\right)$, to be taken as an *algebraic* language translation of $(\mathbb{Q}_{jcn}, +, \cdot)$, is a \mathbb{Z} -module and a \mathbb{Q} -vector plane said to be of *NJIKI*, and the *null* vector therein being $\frac{0}{0}$!!! And *the* one *null* vector $\frac{0}{0}$ being referred to as the *NJIKI's null* vector whereas the *elements* or *items* of \mathbb{Q} are here called *non null* vectors and as well as *those* of $\left\{\frac{b}{0} / b \in \mathbb{Z}^*\right\}$.

The *mathematical* set $\mathbb{Q} \cup \left\{\frac{b}{0} / b \in \mathbb{Z}\right\}$, marked \mathbb{Q}_{nve} , is *named* or *called* the *NJIKI's vector* extension of \mathbb{Q} . Its *extension* in \mathbb{R} and \mathbb{C} are respectively marked \mathbb{R}_{nve} and \mathbb{C}_{nve} . And to *summarize*: $\mathbb{Q}_{nve} = \mathbb{Q} \cup \left\{\frac{b}{0} / b \in \mathbb{Z}\right\}$, $\mathbb{R}_{nve} = \mathbb{R} \cup \left\{\frac{b}{0} / b \in \mathbb{R}\right\}$ and $\mathbb{C}_{nve} = \mathbb{C} \cup \left\{\frac{b}{0} / b \in \mathbb{C}\right\}$. Besides, the common canonical basis of $(\mathbb{Q}_{nve}, +_{1,1}, \bullet)$, $(\mathbb{R}_{nve}, +_{1,1}, \bullet)$, $(\mathbb{C}_{nve}, +_{1,1}, \bullet)$ is $B = \left(\frac{1}{0}, \frac{0}{1}\right)$. co-tangentially, or, $B = \left(\frac{0}{1}, \frac{1}{0}\right)$ tangentially. Making it so be as follows:

$$\forall \frac{b}{a} \in \mathbb{Q}_{nve} \vee \forall \frac{b}{a} \in \mathbb{R}_{nve} \vee \forall \frac{b}{a} \in \mathbb{C}_{nve}, \frac{b}{a} = b \bullet \frac{1}{0} +_{1,1} a \bullet \frac{0}{1} \vee \frac{d}{c} = d \bullet \frac{1}{0} +_{1,1} c \bullet \frac{0}{1} \vee \frac{f}{e} = f \bullet \frac{1}{0} +_{1,1} e \bullet \frac{0}{1}.$$

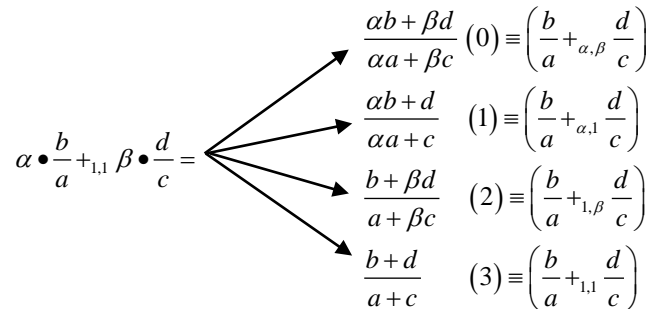
3) $\alpha \frac{b}{a} +_{1,1} \beta \frac{d}{c} = \frac{\alpha b + \beta d}{\alpha a + \beta c}$, by *linear* combination. And as *remark*, that same result, $\frac{\alpha b + \beta d}{\alpha a + \beta c}$, obtained by *linear* combination, can also be achieved by a single

one *additive* operation $+_{\alpha, \beta}$, in \mathbb{Q} , and as follows $\frac{b}{a} +_{\alpha, \beta} \frac{d}{c} = \frac{\alpha b + \beta d}{\alpha a + \beta c}$.

Meaning that the *additive* operation $+_{\alpha, \beta}$, in \mathbb{Q} , can be taken as bearing a *linear* combination affected with *coefficients* α and β . Another major *remark* is that, by definition, the *linear* combination $\alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c}$ has 3 other versions, options, alternatives or possibilities of calculation and that so because of the 2

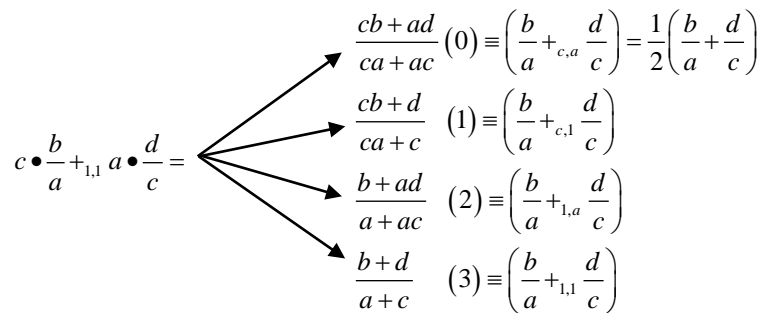
equalities $\frac{\alpha b}{\alpha a} = \frac{b}{a}$ and $\frac{\beta d}{\beta c} = \frac{d}{c}$ leading to 2×2 ways of computing

$\alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c}$, and those 3 other versions are: a) $\alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c} = \frac{\alpha b + d}{\alpha a + c}$, b) $\alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c} = \frac{b + \beta d}{a + \beta c}$ and c) $\alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c} = \frac{b + d}{a + c}$, respectively corresponding to $+_{\alpha,1}$, $+_{1,\beta}$ and $+_{1,1}$. And, diagrammatically:



And the *middle* ones therein and that is to say (1) and (2) being close to some *modular forms*.

EXAMPLE



EXERCISE 1

Make a short *mathematical study*, from the above example, of the *six* (6) equalities between (0), (1), (2) and (3), and 2 by 2. And that so, both in the field

$$\left(\mathbb{Q}, +, \times \right) \text{ and the NJIKTs } \mathbb{Q}\text{-vector plane } \left(\mathbb{Q} \cup \left\{ \frac{b}{0} / b \in \mathbb{Z} \right\}, +_{1,1}, \bullet \right).$$

Conclusion: The *linear combination* $\alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c}$ within the NJIKTs \mathbb{Q} -vector plane $\left(\mathbb{Q} \cup \left\{ \frac{b}{0} / b \in \mathbb{Z} \right\}, +_{1,1}, \bullet \right)$, in terms of the *set theory*, can then be considered as follows:

$$\begin{aligned} \alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c} \in \left\{ \alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c} \right\} &= \left\{ \frac{\alpha b + \beta d}{\alpha a + \beta c}, \frac{\alpha b + d}{\alpha a + c}, \frac{b + \beta d}{a + \beta c}, \frac{b + d}{a + c} \right\} \\ &= \left\{ \frac{b}{a} +_{\alpha,\beta} \frac{d}{c}, \frac{b}{a} +_{\alpha,1} \frac{d}{c}, \frac{b}{a} +_{1,\beta} \frac{d}{c}, \frac{b}{a} +_{1,1} \frac{d}{c} \right\} \end{aligned}$$

And it is called or named, for the *purpose* of the *cause*, “the NJIKTs 4 elements of 1 *number-set* or *set-number*” including mixed up 2 modular forms, $+_{\alpha,1}$ and $+_{1,\beta}$, 1 addition and also found in the hyperbolic geometry, $+_{1,1}$, another 1 addition, $+_{\alpha,\beta}$, specially found in DOCIMOMETRICS, for some de-

tailed description about it, and *its* items are equals 2 by 2 under *certain* circumstances and particularly and in all cases under those of the \mathbb{Q} equivalence classes or classes of equivalence.

And, to make *it*, the story, turn into an image *it* is as if “the *NJIKI*’s 4 elements of 1 *number-set* or *set-number*”, by its *linear* combination $\alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c}$, in $(\mathbb{Q}_{nve}, +_{1,1}, \bullet)$, were gathering into the same toolbox or *dissection* kit, etc., the 4 *addictive* operations, in \mathbb{Q} , $+_{\alpha,\beta}$, $+_{\alpha,1}$, $+_{1,\beta}$, and $+_{1,1}$. And that *last* one $+_{1,1}$, *generalizing* the 4 other equal *addictive* operations $=_{\alpha,\beta}$, $=_{\alpha,1}$, $=_{1,\beta}$, and $=_{1,1}$ within the \mathbb{Q} *equivalence* classes or *classes* of equivalence, being as a *generator* at the *theoretical* basis of the construction of all others including even the *number-set* or *set-number* itself, $\alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c}$, in $(\mathbb{Q}_{nve}, +_{1,1}, \bullet)$. And that so, in addition to both its *natural* and *granulometric* character and its use, *by default*, when it comes to get involved into applications.

EXERCISE 2

Show the following said to be the two mathematics’ coachings or the two double inequalities of *NJIKI*:

- 1) $\forall \left(\frac{b}{a}, \frac{d}{c}\right) \in \mathbb{Q}^{+2} \wedge \forall (\alpha, \beta) \in \mathbb{Q}^{+*2}, \frac{b}{a} \leq \frac{d}{c} \Rightarrow \frac{b}{a} \leq \alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c} \leq \frac{d}{c}$
- 2) $\forall \left(\frac{b}{a}, \frac{d}{c}\right) \in \mathbb{Q}^{+2} \wedge \forall (\alpha, \beta) \in \mathbb{Q}^{+*2}, \frac{b}{a} < \frac{d}{c} \Rightarrow \frac{b}{a} < \alpha \bullet \frac{b}{a} +_{1,1} \beta \bullet \frac{d}{c} < \frac{d}{c}$

Note:

1) a fraction is definitely both a number, said to be relative according to *NJIKI* and as explained below, in Part (C), and not rational as usual, and, a vector.

$$2) \forall \left(\frac{b}{a}, \frac{d}{c}\right) \in \mathbb{Q}^{+2}, \frac{b}{a} < \frac{b}{a} + \frac{d}{c} \wedge \frac{d}{c} < \frac{b}{a} + \frac{d}{c}$$

EXERCISE 3, or, the QUIZ that makes it talk about Mathematics as the 1st Natural Sciences among all those of that category.

Say it practically physically how to interpret the theoretical weighting coefficients λ and μ in the 1 *number-set* or *set-number* $\left\{ \lambda \bullet \frac{b}{a} +_{1,1} \mu \bullet \frac{d}{c} \right\}$. Justify your answer (and the which answer having a very big interest in the PART (C) below).

ANSWER PROPOSAL

They, λ and μ in the 1 *number-set* or *set-number* $\left\{ \lambda \bullet \frac{b}{a} +_{1,1} \mu \bullet \frac{d}{c} \right\}$, practically physically can, and for the sake of various and numerous applications, be referred to as the adjustment or conversion coefficients into the same physical units or scales between those common to a and b , and, those other common to c and d . And by so doing, for the justification now, in order to enable the 2 factorisations between both the b and d level, and, the a and c one, being by the same unit or scale term that can then be simplified or not.

EXERCISE 4: **Fractions** and *Theories* of **Representation**

For the tip about it, this EXERCISE 4 is a conclusive exercise consisting in a

comparative practical study between the classical or traditional operation of addition + and that carried out by the NJIKI's 4 elements of 1 number-set or set-number $\alpha \cdot \frac{b}{a} +_{1,1} \beta \cdot \frac{d}{c} \in \left\{ \alpha \cdot \frac{b}{a} +_{1,1} \beta \cdot \frac{d}{c} \right\}$, in \mathbb{Q} . Knowing that from $\frac{b}{a} = \frac{\lambda b}{\lambda a}$ and $\frac{d}{c} = \frac{\mu d}{\mu c}$, 1) $\frac{b}{a} + \frac{d}{c} = \frac{bc + ad}{ac}$, 2) $\frac{\lambda b}{\lambda a} + \frac{\mu d}{\mu c} = \frac{bc + ad}{ac}$, 3) $\frac{\lambda b}{\lambda a} + \frac{d}{c} = \frac{bc + ad}{ac}$, and 4) $\frac{b}{a} + \frac{\mu d}{\mu c} = \frac{bc + ad}{ac}$; and what shows the independence of $\frac{b}{a} + \frac{d}{c}$ upon λ and μ . And the final step of it being into how $\frac{b}{a} + \frac{d}{c}$ and $\alpha \cdot \frac{b}{a} +_{1,1} \beta \cdot \frac{d}{c}$ can be viewed from their splitting into proportional segments or sectors on or in a straight line or a disc representation in a plane, for example. By answering to the question whether $\frac{b}{a} + \frac{d}{c}$ can make sense by extracting $\frac{b}{a}$ and $\frac{d}{c}$ from two different physical items, of the same nature, and of course, or not. And idem with $\alpha \cdot \frac{b}{a} +_{1,1} \beta \cdot \frac{d}{c}$. The ultimate aim of such an exercise being to set out the existence of constraints or conditions or not in the practical use of $\frac{b}{a} + \frac{d}{c}$ or of $\alpha \cdot \frac{b}{a} +_{1,1} \beta \cdot \frac{d}{c}$. And beyond over that, the establishment of the existence of the eventual limits of mathematics, and the mathematics remaining much more formal than meaningful.

And now, practice or exercise with $\frac{2}{5} + \frac{1}{3}$ and $\lambda \cdot \frac{2}{5} +_{1,1} \mu \cdot \frac{1}{3}$ in the following 3 arbitrarily chosen cases: 1) $\lambda = \mu$, by default, 2) $\lambda = 10\mu$ and 3) $\lambda = 3\mu$. Else $\lambda \cdot \frac{2}{5} +_{1,1} \mu \cdot \frac{1}{3}$ remains once at least or twice parametric and in λ or μ . So then, and that being said, shift to the generalization of the above with: 1) $\frac{b}{a} + \frac{d}{c} = \frac{bc + ad}{ac}$ opening way to 3 scales of ac, and especially in the case that $ac > bc + ad$, namely a, c and ac, and, 2) $\lambda \cdot \frac{b}{a} +_{1,1} \mu \cdot \frac{d}{c}$, setting $\lambda = k\mu$ (1) and where λ and μ are respective proportional units of $\frac{b}{a}$ and $\frac{d}{c}$ in a certain physical system of units. And by setting either $\lambda = 1$ or (e) $\mu = 1$ (2), one moves towards modular forms. Meaning that the modular forms hold their origin into the NJIKI's 4 elements of 1 number-set or set-number which themselves derive from the NJIKI's Fundamental Theorem-Definition... In other words or terms, the beginning of the major items of the modular forms is at the prolongation of the NJIKI's Fundamental Theorem-Definition..., and that through the NJIKI's 4 elements of 1 number-set or set-number. And what makes so then another much more merit to the ongoing paper or article, to salute or celebrate and because it got jumbled over and straight to the study of the modu-

lar forms depending upon it, in relation to the natural and methodic order of bringing into construction the mathematical stuffs. And the same way of reasoning can be apply as concerning the hyperbolic geometry.

(C) The *special* idea behind the *NJIKI's* proposal about the *logical* exchange, interchange, reverse or permutation of *names* by predicates between the current “*relative*” integers of \mathbb{Z} and the “*rational*” fractions or numbers of \mathbb{Q} , and, *special* idea of *reverse* or permutation of names to make it be reviewed by *other* mathematicians around the world and as the opening of the debate.

The “*relative*” integers of \mathbb{Z} , according to *NJIKI*, will henceforth become the “*rational*” integers of \mathbb{Z} still, and, the “*rational*” fractions or numbers of \mathbb{Q} the “*relative*” fractions or numbers of \mathbb{Q} . And all *things* of “*relative*”, *linked* to \mathbb{Q} , due to its *equivalence* classes or *classes* of equivalence, $\frac{b}{a} = \frac{kb}{ka}$ with $k \in \mathbb{Z}^*$, and which can be said to be of a *multiplicative* type, giving *relative* choice of *possibilities* in the use of fractions of the same *equivalence* class or *class* of equivalence, and *fractions* of the same *equivalence* class or *class* of equivalence called *its* representatives and indeed *its* relative representatives constituting the heart of the announced debate in the title, and according to the *problem* cases to go through, as in the *arithmetic* ones. For example, to *make* $\frac{b}{a} + \frac{d}{c}$, one has to use $\frac{bc}{ac}$ and $\frac{ad}{ac}$ and then *two* other fractions, *relative* to the *first* ones $\frac{b}{a}$ and $\frac{d}{c}$, and *they* themselves resulting from an infinite *system*

of units as follows: $\frac{b}{a} = b \cdot a^{-1} = \frac{b}{c^{-1}} \cdot a^{-1}c^{-1} = bc \cdot (ac)^{-1} = \frac{bc}{ac}$ and

$\frac{d}{c} = d \cdot c^{-1} = ad \cdot \frac{c^{-1}}{a} = ad \cdot a^{-1}c^{-1} = ad \cdot (ac)^{-1} = \frac{ad}{ac}$.

And finally, by *factorization* $\frac{bc}{ac} + \frac{ad}{ac} = \frac{1}{ac}(bc + ad) = \frac{bc + ad}{ac} = \frac{b}{a} + \frac{d}{c}$, or,

$bc \cdot (ac)^{-1} + ad \cdot (ac)^{-1} = (bc + ad)(ac)^{-1} = \frac{bc + ad}{ac} = \frac{b}{a} + \frac{d}{c}$.

In other words, it is as if to convert the *numerical* unit a^{-1} of b in $\frac{b}{a} = b \cdot a^{-1}$ and *that* c^{-1} of d in $\frac{d}{c} = d \cdot c^{-1}$ into the *same* unit $a^{-1}c^{-1} = (ac)^{-1}$, of a *numerical* nature of course but *not* different from *those* found in physics and justifying much more the *predicate* “*relative*” than will be doing it the *integers* of \mathbb{Z} , in order to make the *addition* $bc(ac)^{-1} + ad(ac)^{-1}$ possible by the way of the *factorisation*: $bc(ac)^{-1} + ad(ac)^{-1} = (bc + ad)(ac)^{-1}$.

The *equivalence* classes or the *classes* of equivalence of \mathbb{Z} , which can be said to be of an *additive* type, do not show or enjoy such a *property* of relativity in terms of an *infinite system* of units in multiples and *sub-multiples*.

From the above, the *epithet*-adjective “*relative*” goes best with the fractions or numbers of \mathbb{Q} and not so much with the integers of \mathbb{Z} receiving the *epi-*

thet-adjective “rational”, by exchange or interchange, without problem.

Besides, the *elements* of \mathbb{Q}_{jcn} are named the *NJIKI*'s vectors, fractions or numbers.

And by the way, the *major* interest of the paper here is to *find*, and of course within the *respect* of the copyright, “the *NJIKI*'s fundamental THEOREM-DEFINITION on fractions in the *mathematical* set \mathbb{Q} and by *extension* in \mathbb{R} and \mathbb{C} ” and its related stuffs into the *teaching* and the *learning* mathematical books all over the world and where they should be thought and learned in schools and applied at *work* or *job* places and specially in the areas of *computer* sciences by supplying them with new and *specific* methods of *Fractions*' computation.

And now, and to make it by the way of an image, what will then henceforth be named as *relative* fractions or numbers, of \mathbb{Q} , were in the past like an *one-foot* being and then *unable* to walk. They have been given another foot, by “the *NJIKI*'s fundamental THEOREM-DEFINITION on fractions in the *mathematical* set \mathbb{Q} and by *extension* in \mathbb{R} and \mathbb{C} ”, in order to make them *be* walking, in algebra and in calculus as well. And their *various* and *numerous* applications in every day life of the human beings being not to be put apart or aside. And *they* are so *relative*, and as for one much more reason, because *they* can be used either by one of their *two* feet, to count, either by another, to measure. And that is another heart issue of the *property* of the relativity, and rather than *that* of the rationality, attached to the *fractions* or *numbers* of \mathbb{Q} , as the *field* $(\mathbb{Q}, +, \times)$, and the *fractions* or *numbers* of \mathbb{Q} which are also *recognized* now to be vectors in the *NJIKI*'s \mathbb{Z} -module $\left(\mathbb{Q} \cup \left\{ \frac{b}{0} / b \in \mathbb{Z} \right\}, +_{1,1}, \bullet \right)$ or \mathbb{Q} -vector plane

$$\left(\mathbb{Q} \cup \left\{ \frac{b}{0} / b \in \mathbb{Z} \right\}, +_{1,1}, \bullet \right).$$

In the *final* conclusion, the *predicate* “rational” as or *meaning* logical does go very well both for the *integers* of \mathbb{Z} and the *fractions* or *numbers* of \mathbb{Q} but the *adjective* “relative” goes *best* for the *fractions* or *numbers* of \mathbb{Q} and not for the *integers* of \mathbb{Z} . And then henceforth the *rational* integers of \mathbb{Z} and the *relative* fractions or numbers of \mathbb{Q} which have to be used in mathematics instead of the *current* *relative* integers of \mathbb{Z} and the *rational* fractions or numbers of \mathbb{Q} .

More of facts, in calculus and much more than in algebra, a *ratio*, $\frac{b}{a}$ or $\frac{d}{c}$, and particularly when $\frac{b}{a} \neq \frac{d}{c} \wedge \frac{b-d}{a-c} = \frac{d-b}{c-a} \neq \frac{b}{a}$, is usually referred to as a *relative* value as compared to *those* of the *integers* a, b, c or d constituting it and said to be the *absolute* ones. And what is very used in physics in *calculating* some sorts or kinds of *relative* variations in *physical* measures at a certain *margin* of error and although there is another *relation* of *relative* value to *absolute* value between the *items* of \mathbb{Z} and *those* of \mathbb{N} , and from the algebra's *view* point. And it will then be welcome to get things harmonized from the *mathe-*

matical vocabulary's *point* of view.

And, after my above *opening* remarks, *the floor is* now open.

So then, thank you very much for your reading, ladies and gentlemen, dear readers and researchers in mathematics, and while waiting for your kind numerous and various opinions or reactions to the topic.