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Adaptive Robust Control with Leakage-Type Control Law for Trajectory Tracking of Exoskeleton Robots

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Abstract

This paper investigates the trajectory following problem of exoskeleton robots with numerous constraints. However, as a typical nonlinear system with variability and parameter uncertainty, it is difficult to accurately achieve the trajectory tracking control for exoskeletons. In this paper, we present a robust control of trajectory tracking control based on servo constraints. Firstly, we consider the uncertainties (e.g., modelling errors, initial condition deviations, structural vibrations, and other unknown external disturbances) in the exoskeleton system, which are time-varying and bounded. Secondly, we establish the dynamic model and formulate a close-loop connection between the dynamic model and the real world. Then, the trajectory tracking issue is regarded as a servo constraint problem, and an adaptive robust control with leakage-type adaptive law is proposed with the guaranteed Lyapunov stability. Finally, we conduct numerical simulations to verify the performance of the proposed controller.

Keywords

Trajectory Tracking, Adaptive Robust Control, Exoskeleton Robots, Uncertainties

1. Introduction

In recent years, with the intensification of the aging problem, the labour force has become increasingly scarce [1]-[3]. The number of patients with limb dysfunction has also been continuously rising, primarily due to stroke, car accidents, and other unexpected incidents [4]-[6]. Additionally, with the advancement of science and technology, people are placing greater emphasis on health and work efficiency [7]-[10].

Exoskeleton robots have garnered significant attention due to their effectiveness in areas such as rehabilitation training, walking assistance, and physical enhancement [11]-[13]. Trajectory tracking control is the foundation for achieving advanced control in exoskeletons, which is the focus of this paper. However, the nonlinear characteristics and constraints of exoskeleton systems present challenges for precise control.

To achieve accurate control performance, scholars in the field have attempted many methods in recent years, such as impedance control [14], force control [15], position control [16], hybrid force/position control [17], and fuzzy control [18]. Sun *et al.* achieved a hardware-independent safety-focused admittance control approach to guarantee the user's safety in the control process [19]. Wu *et al.* developed a fuzzy adaptive admittance control based on a neural network compensator for exoskeletons to overcome the uncertainties of dynamic modelling [20]. Wang *et al.* proposed a hierarchical structure with PI^2-based adaptive impedance control to track the desired trajectory [21]. The spatial iterative learning torque control was designed to achieve better performance with high control accuracy for exoskeleton robots [22]. To improve the position control stability and accuracy of exoskeletons, Yin *et al.* emphasized the importance of considering nonlinear factors in system dynamics modelling and utilized BP neural networks to adjust control parameters in real time for implementing a rotation angle control strategy [23]. Li *et al.* presented a saturated sliding mode control strategy and introduced an extended state observer to estimate system uncertainties and disturbances; finally, an integral sliding mode observer compensated for trajectory tracking errors, enhancing tracking performance [24]. Chang *et al.* proposed a two-layer control structure based on a Lyapunov-based switched systems approach: a high-level controller with repetitive learning to follow the desired trajectories and a low-level controller with admittance models to adjust the cable tension [25]. Huang *et al.* designed a fuzzy enhanced adaptive admittance control to consider human-machine interaction forces to track a reference trajectory, and the effectiveness of the tracking performance was validated through experiments [26]. To solve the unstable problem in the exoskeleton system, a typically multi-input-multi-output uncertain nonlinear system, Sun *et al.* reported a reduced adaptive fuzzy decoupling control scheme with a compensation term [27]. Li *et al.* provided a novel control strategy based on zeroing dynamics to minimize potential energy variation and achieve accurate motion control of exoskeletons [28]. Chen *et al.* proposed an active disturbance rejection control equipped with fast terminal sliding mode control to enhance the trajectory tracking performance while alleviating the external disturbance [29].

The aforementioned control methods all rely on linearization and approximation to construct the system model and achieve their respective objectives. However, this is difficult to accomplish in nonlinear systems, especially when the system becomes complex. Existing methods for formulating dynamic models are based on traditional classical mechanics (such as Lagrangian mechanics), which makes it difficult to determine variables and increases computational costs in typical nonlinear systems like exoskeletons [30]-[32].

To address the existing problems, we approach Lagrangian dynamics from a different perspective, constructing explicit general motion equations for constrained nonlinear exoskeleton systems and establishing the connection between constraints and the dynamic model. In addition, the uncertainties that are inevitable and unknown in real practice are considered in the proposed control method. Our main contributions are as follows:

1) We establish the exoskeleton mechanical system description subject to constraints by closed-form constrained motion equation, bridging a connection between the model and constraints.

2) We design an adaptive robust control with leakage-type control law under uncertainties to improve the trajectory tracking performance while ensuring Lyapunov stability.

3) We perform numerical simulations to demonstrate the effectiveness of the proposed control method.

The paper is structured as follows. Section 2 introduces the exoskeleton mechanical system description. Section 3 describes the adaptive robust control with leakage-type control law. Numerical simulations are conducted in Section 4, followed by a conclusion of the full paper.

2. Exoskeleton Mechanical System Description

We base our work on a lower limb exoskeleton and establish a dynamic model for the exoskeleton mechanical system under constrained conditions. The flexion and extension degrees of freedom of the hip and knee joints of the lower limb exoskeleton are active degrees of freedom, while the remaining degrees of freedom are passive. **Figure 1** demonstrates the simplified model of the exoskeleton single leg, which contains a thigh and shank-foot. **Table 1** presents the parameters included in the simplified model of the exoskeleton single leg.

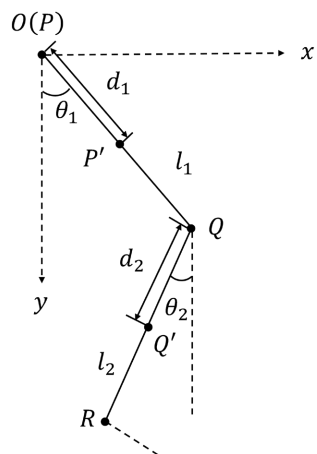

Figure 1. The simplified model of the exoskeleton single leg.

Table 1. The parameters of the exoskeleton single leg.

Contents	Parameters	Units
$O(P)$	Hip joint (Coordinate origin)	/
Q	Knee joint	/
R	Ankle joint	/
P'	Center of mass of thigh	/
Q'	Center of mass of shank	/
$\theta_i\ (i=1,2)$	Angle between the i-th linkage and the Y-axis	rad
m_1	Mass of the thigh	kg
m_2	Mass of the shank	kg
l_1	Length of the thigh	m
l_2	Length of the shank	m
d_1	The length between the center of mass of the 1-th part and hip joint	m
d_2	The length between the center of mass of the 2-th part and the knee joint	m
I_1	Moment of inertia of the thigh	kg·m^2
I_2	Moment of inertia of the shank	kg·m^2
g	Gravitational acceleration	m/s^2

The unconstrained dynamic model of the exoskeleton can be described as

$$M(q,t)\ddot{q}+C(q,\dot{q},t)+G(q,t)=0 \tag{1}$$

where

$$M(q,t)=\begin{bmatrix} m_1d_1^2+I_1+m_2l_1^2 & -m_2l_1d_2\cos(\theta_1+\theta_2) \\ -m_2l_1d_2\cos(\theta_1+\theta_2) & m_2d_2^2+I_2 \end{bmatrix} \tag{2}$$

$$C(q,\dot{q},t)=\begin{bmatrix} m_2l_1d_2\sin(\theta_1+\theta_2)\dot{\theta}_2 & -m_2l_1d_2\sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) \\ -m_2l_1d_2\sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) & m_2l_1d_2\sin(\theta_1+\theta_2)\dot{\theta}_1 \end{bmatrix} \tag{3}$$

$$G(q,t)=\begin{bmatrix} -m_1gd_1\sin\theta_1-m_2gl_1\sin\theta_1 \\ -m_2gd_2\sin\theta_2 \end{bmatrix} \tag{4}$$

$$F(q,\dot{q},t)=-\big(C(q,\dot{q},t)+G(q,t)\big). \tag{5}$$

But in the real scenarios, the constraints in the system are inevitable. We formulate constraints in mechanical systems as follows

$$\sum_{i=1}^{n}B_{li}(q,t)=d_l(q,t),\ \ l=1,\cdots,m. \tag{6}$$

Equation (6) can be transferred to the matrix form

$$B(q,t)=d(q,t) \tag{7}$$

where $B=[B_{li}]_{m\times n}$, $d=[d_1,d_2,\cdots,d_m]^{\mathrm{T}}$.

By taking the first derivative of equation (3) concerning *t*, equation (8) can be obtained

$$\sum_{i=1}^{n} A_{li}(q,t)\dot{q}=c_l(q,t) \tag{8}$$

where

$$A_{li}(q,t)=\sum_{k=1}^{n}\frac{\partial B_{li}(q,t)}{\partial q_k}-\sum_{k=1}^{n}\frac{\partial d_l(q,t)}{\partial q_k}, \tag{9}$$

$$c_l(q,t)=\frac{\partial d_l(q,t)}{\partial t}-\frac{\partial B_{li}(q,t)}{\partial t}. \tag{10}$$

Rewrite the constraints (8) as a matrix form:

$$A(q,t)\dot{q}=c(q,t) \tag{11}$$

where $A=[A_{li}]_{m\times n}$ and $c=[c_1,c_2,\cdots,c_m]^{\mathrm{T}}$ which contains the first-order derivative of q.

The second-order derivative of equation (8) can be expressed:

$$\sum_{i=1}^{n} A_{li}(q,t)\ddot{q}=b_l(q,\dot{q},t) \tag{12}$$

where

$$b_l(q,\dot{q},t)=\frac{\mathrm{d}}{\mathrm{d}t}c_l(q,t)-\sum_{i=1}^{n}\left(\frac{\mathrm{d}}{\mathrm{d}t}A_{li}(q,t)\right)\dot{q}_i \tag{13}$$

which can be transferred to:

$$A(q,t)\ddot{q}=b(q,\dot{q},t) \tag{14}$$

where $b=[b_1,b_2,\cdots,b_m]^{\mathrm{T}}$, which contains the second-order derivative of q.

Assume the unconstrained dynamic model (1) is subject to the constraints (14), the constrained mechanical system can be formulated as:

$$M(q,t)\ddot{q}=F(q,\dot{q},t)+F^c(q,\dot{q},t). \tag{15}$$

Based on descriptions in [33], the $F^c(q,\dot{q},t)\in R^n$ can be expressed by:

$$\begin{aligned}F^c(q,\dot{q},t)=&M^{\frac{1}{2}}(q,t)\left[A(q,t)M^{-\frac{1}{2}}(q,t)\right]^{+}\\&\times\left[b(q,\dot{q},t)-A(q,t)M^{-1}(q,t)F(q,\dot{q},t)\right]\end{aligned} \tag{16}$$

where "+" denotes the Moore-Penrose (MP) inverse.

3. Adaptive Robust Control

In typical nonlinear systems like exoskeletons, uncertainties are unavoidable, such as initial condition deviations, human-machine interaction forces, and external disturbances. Therefore, we consider uncertainties in the constrained mechanical system:

$$M(q,\sigma,t)\ddot{q}+C(q,\dot{q},\sigma,t)\dot{q}+G(q,\sigma,t)=\tau \tag{17}$$

where $\sigma\in\Sigma\subset R^p$ is time-varying and bounded (but unknown).

Separate the matrices/vectors *M*, *C*, and *G* into two distinct components: the "nominal" components and the "uncertain" components, as illustrated below:

$$M(q,\sigma,t)=\bar{M}(q,t)+\Delta M(q,\sigma,t), \tag{18}$$

$$C(q,\dot{q},\sigma,t)=\bar{C}(q,\dot{q},t)+\Delta C(q,\dot{q},\sigma,t), \tag{19}$$

$$G(q,\sigma,t)=\bar{G}(q,t)+\Delta G(q,\sigma,t). \tag{20}$$

Assume that $\bar{M}>0$. $\bar{M}(\cdot)$, $\bar{C}(\cdot)$, $\bar{G}(\cdot)$, $\Delta M(\cdot)$, $\Delta C(\cdot)$ and $\Delta G(\cdot)$ are all continuous. In the ideal case, the "uncertain" components are equal to zero. Let

$$E(q,\sigma,t):=\bar{M}(q,t)M^{-1}(q,\sigma,t)-I, \tag{21}$$

$$D(q,t):=\bar{M}^{-1}(q,t). \tag{22}$$

Assumption 1: For each $(q,t)\in R^n\times R$, $A(q,t)$ is of full rank, which means that $A(q,t)A^{\mathrm{T}}(q,t)$ is invertible.

Assumption 2: When *Assumption* 1 is satisfied, for a given $P\in R^{m\times m}$, $P>0$, let

$$\begin{aligned}W(q,\sigma,t):=&PA(q,t)D(q,t)E(q,\sigma,t)\bar{M}(q,t)\\&\times A(q,t)\left(A(q,t)A^{\mathrm{T}}(q,t)\right)^{-1}P^{-1}.\end{aligned} \tag{23}$$

For all $(q,t)\in R^n\times R$, there exists a constant $\rho_E>-1$ satisfies the equation (24):

$$\frac{1}{2}\min\left(\lambda_m\left(W(q,\sigma,t)+W^{\mathrm{T}}(q,\sigma,t)\right)\right)\geq\rho_E \tag{24}$$

where λ_m denotes the eigenvalues of the matrix.

The value of ρ_E, *W*, and *E* depend on *W*, *E*, and the boundary of the uncertainties, respectively. Since the uncertainties have an unknown boundary, ρ_E is unknown. In the ideal situation, $E=0$, $W=0$, and $\rho_E=0$.

Let $\Delta D(q,\sigma,t):=M^{-1}(q,\sigma,t)-\bar{M}(q,t)$, then

$$\Delta D(q,\sigma,t)=D(q,t)E(q,\sigma,t). \tag{25}$$

Assumption 3: 1) There exists an unknown constant vector $\alpha\in(0,\infty)^k$ and a known function $\Pi(\cdot):(0,\infty)^k\times R^n\times R^n\times R\to R_+$, such that, for all $(q,\dot{q},t)\in R^n\times R^n\times R$, $\sigma\in\Sigma$,

$$\begin{aligned}&(1+\rho_E)^{-1}\max\Big(\Big\|PA(q,t)\Delta D(q,\sigma,t)\big(-C(q,\dot{q},\sigma,t)\\&-G(q,\sigma,t)+w_1(q,\dot{q},t)+w_2(q,\dot{q},t)\big)-PA(q,t)\\&\times D(q,t)\big(\Delta C(q,\dot{q},\sigma,t)\dot{q}+\Delta G(q,\sigma,t)\big)\Big\|\Big)\leq\Pi(\alpha,q,\dot{q},t).\end{aligned} \tag{26}$$

2) For each $(\alpha,q,\dot{q},t)$, $\Pi(\alpha,q,\dot{q},t)$ is a linearized function with respect to α; there exists a function $\Pi(\cdot):R^n\times R^n\times R\to R_+$ such that

$$\Pi(\alpha,q,\dot{q},t)=\alpha^{\mathrm{T}}\tilde{\Pi}(q,\dot{q},t). \tag{27}$$

Here, the unknown boundary of the uncertainties Σ determines the value of α.

According to equation (16) and *Assumption* 1 - 3, the adaptive robust control $\tau(t)$ can be formulated as follows:

$$\tau(t)=w_1(q(t),\dot{q}(t),t)+w_2(q(t),\dot{q}(t),t)+w_3(q(t),\dot{q}(t),t) \tag{28}$$

where

$$\begin{aligned}w_1(q,\dot{q},t)&:=\bar{M}^{\frac{1}{2}}(q,t)\left(A(q,t)\bar{M}^{-\frac{1}{2}}(q,t)\right)^{+}\big(b(q,\dot{q},t)\\&\quad-A(q,t)\bar{M}^{-1}(q,t)F(q,\dot{q},t)\big)\end{aligned} \tag{29}$$

$$w_2(q,\dot{q},t):=-\kappa\bar{M}(q,t)A^{\mathrm{T}}(q,t)\left(A(q,t)A^{\mathrm{T}}(q,t)\right)^{-1}\times P^{-1}\beta(q,\dot{q},t) \tag{30}$$

$$w_3(q,\dot{q},t):=-\bar{M}(q,t)A^{\mathrm{T}}(q,t)\left(A(q,t)A^{\mathrm{T}}(q,t)\right)^{-1}\times P^{-1}\chi(\hat{\alpha},q,\dot{q},t) \tag{31}$$

$$\chi(\hat{\alpha},q,\dot{q},t)=\gamma(\hat{\alpha},q,\dot{q},t)\mu(\hat{\alpha},q,\dot{q},t)\Pi(\hat{\alpha},q,\dot{q},t) \tag{32}$$

$$\gamma(\hat{\alpha},q,\dot{q},t)=\begin{cases}\dfrac{1}{\left\|\mu(\hat{\alpha},q,\dot{q},t)\right\|} & \text{if } \left\|\mu(\hat{\alpha},q,\dot{q},t)\right\|>\hat{\varepsilon}\\ \dfrac{1}{\hat{\varepsilon}} & \text{if } \left\|\mu(\hat{\alpha},q,\dot{q},t)\right\|>\hat{\varepsilon}\end{cases} \tag{33}$$

where $\kappa\in R$, $\kappa>0$; $\hat{\varepsilon}>0$ is a scalar.

$$\mu(\hat{\alpha},q,\dot{q},t)=\beta(q,\dot{q},t)\Pi(\hat{\alpha},q,\dot{q},t) \tag{34}$$

$$\beta(q,\dot{q},t)=A(q,t)\dot{q}-c(q,t)+\eta\left(B(q,t)-d(q,t)\right) \tag{35}$$

where $B(q,t)-d(q,t)$ denotes the constraint error and η indicates its weight; $A(q,t)\dot{q}-c(q,t)$ stands for the constrained first-order error.

The leakage-type control law can be formulated to govern the parameter $\hat{\alpha}$ as follows:

$$\dot{\hat{\alpha}}=k_1\tilde{\Pi}(q,\dot{q},t)\left\|\beta(q,\dot{q},t)\right\|-k_2\hat{\alpha} \tag{36}$$

$\dot{\hat{\alpha}}_i(t_0)>0$ (where $\hat{\alpha}_i$ is the *i*-th element of the vector $\hat{\alpha}$, $i=1,2,\cdots,k$), $k_1,k_2\in R^+$. The adaptive parameter $\hat{\alpha}$ is a substitute for the unknown constant α, and adaptive law (38) can determine the value of $\hat{\alpha}$.

Theorem 2: Let $\delta:=\left[\beta^{\mathrm{T}}\quad(\hat{\alpha}-\alpha)^{\mathrm{T}}\right]^{\mathrm{T}}\in R^{m+k}$. Assume that *Assumptions* 1 - 3 are satisfied, under the system of (19), the adaptive robust control (30) renders the UB and UUB:

1) Uniform boundedness: For any $r>0$, there exists a $d(r)<\infty$, such that if $\left\|\delta(t)\right\|\le r$, then $\left\|\delta(t)\right\|\le d(r)$ for all $t\ge t_0$.

2) Uniform ultimate boundedness: For any $r>0$ and $\delta(t_0)\le r$, there exists a $\underline{d}>0$. Such that for any $\bar{d}>\underline{d}$ as $t>t_0+T(\bar{d},r)$, where $T(\bar{d},r)<\infty$, there exists $\delta(t)\le\bar{d}$.

4. Numerical Simulations

Numerical simulations of the proposed control method are shown in this section. Table 2 shows the values of parameters for the exoskeleton system.

Table 2. The values of parameters for the exoskeleton system.

Contents	Parameters	Units
$\bar{m}_1$	36	kg
$\bar{m}_2$	25	kg
l_1	0.5	m
l_2	0.45	m
I_1	2.5	kg·m^2
I_2	1.5	kg·m^2
g	9.8	m/s^2
k_1	1	/
k_2	1	/
κ	1	/
$\hat{\varepsilon}$	0.001	/
$\hat{\alpha}(0)$	0.2	/

4.1. The Desired Trajectory of the Exoskeleton

We chose equations (37) - (38) as the desired trajectory, which is obtained from Clinical Gait Analysis data [33]:

$$\theta_1 = 0.8 + 0.2\sin(2\pi t), \tag{37}$$

$$\theta_2 = 1.2 - 0.2\cos(2\pi t). \tag{38}$$

We rewrite the equations (37) - (38) in the form of the equation (14)

$$A(q,t)\ddot{q} = b(q,\dot{q},t) \tag{39}$$

where

$$A(q,t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{40}$$

$$b = \begin{bmatrix} -0.8\pi^2 \sin(2\pi t) \\ 0.8\pi^2 \cos(2\pi t) \end{bmatrix} \tag{41}$$

The ideal initial conditions are $\theta_1(0) = 0.8$, $\theta_2(0) = 1$, $\dot{\theta}_1(0) = 0.4$, $\dot{\theta}_2(0) = 0$. We consider the uncertainties in the exoskeleton system and chose the initial conditions as $\theta_1(0)=1.2$, $\theta_2(0)=1.2$, $\dot{\theta}_1(0)=0$, $\dot{\theta}_2(0)=0$. The masses are chosen as the uncertain parameters:

$$m_1=\bar{m}_1+\Delta m_1(t), \tag{42}$$

$$m_2=\bar{m}_2+\Delta m_2(t), \tag{43}$$

$$\Delta m_1(t)=0.1\cos(0.1t), \tag{44}$$

$$\Delta m_2(t)=0.05\cos(0.01t). \tag{45}$$

Furthermore, the friction is designed as

$$F_f=A_1\sin(\omega t)+A_2\sin(3\omega t)+A_3\sin(5\omega t) \tag{46}$$

where $A_1=3, A_2=2, A_3=1, \omega=\pi$. If there are no uncertainties in the system, then $\Delta m_1(t)=0$, $\Delta m_2(t)=0$, and $F_f=0$. Finally, the parameter $P=I_{2\times 2}$ is chosen. Then, *Assumptions* 1 - 2 can be easily satisfied. To meet *Assumption* 3, $\Pi(\alpha,q,\dot{q},t)$ can be selected as follows:

$$\Pi(\alpha,q,\dot{q},t)=\alpha_1\|\dot{q}\|^2+\alpha_2\|\dot{q}\|+\alpha_3=\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\end{pmatrix}\begin{bmatrix}\|\dot{q}\|^2\\ \|\dot{q}\|\\ 1\end{bmatrix} \tag{47}$$

where $\alpha_i>0(i=1,2,3)$. To satisfy *Assumption* 3, $\Pi(\alpha,q,\dot{q},t)$ can also be chosen as follows

$$\alpha_1\|\dot{q}\|^2+\alpha_2\|\dot{q}\|+\alpha_3\le\alpha\left(\|\dot{q}\|+1\right)^2=\alpha^{\mathrm{T}}\tilde{\Pi}(q,\dot{q},t) \tag{48}$$

where $\alpha=\max\left\{\alpha_1 \quad \frac{\alpha_2}{2} \quad \alpha_3\right\}$. Based on (47), we can also design the adaptive law as follows

$$\dot{\hat{\alpha}}=k_1\left(\|\dot{q}\|+1\right)^2\|\beta\|-k_2\hat{\alpha} \tag{49}$$

where k_1 and k_2 are constants.

4.2. Results of Numerical Simulations

We verify the proposed control method in both conditions (with or without uncertainties) of the exoskeleton, as shown in **Figures 2-5**. **Figures 2-3** demonstrate the trajectory tracking performance of hip angle θ_1 and its tracking error. Similarly, **Figures 4-5** show the corresponding results of knee angle θ_2. When the system is free of uncertainties, the tracking errors of the hip and knee quickly converge to zero, accompanied by a smaller error amplitude (i.e. maximum error) compared to the condition with uncertainties.

Figure 6 shows the variation of adaptive parameters $\hat{\alpha}$ during trajectory tracking. In both conditions, $\hat{\alpha}$ increases rapidly then drops in a few seconds, and then gradually stabilizes around zero. Compared with the condition without uncertainties, the amplitude of adaptive parameter $\hat{\alpha}$ is larger, but it can also quickly converge to around 0, which further demonstrates the robustness of the proposed control method.

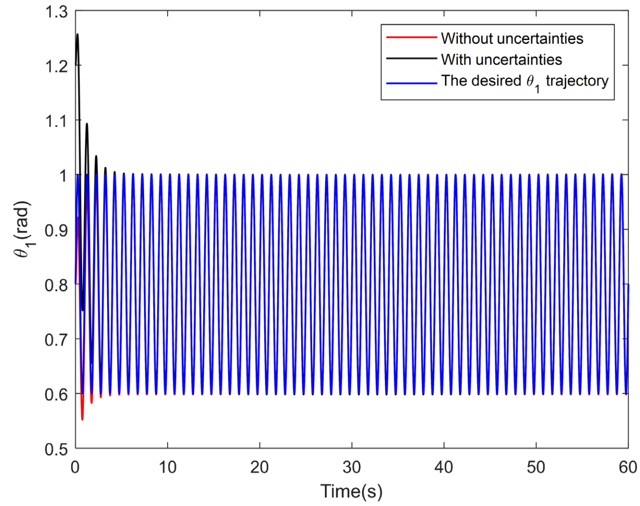

Figure 2. The trajectory tracking performance of hip angle θ_1 in both conditions.

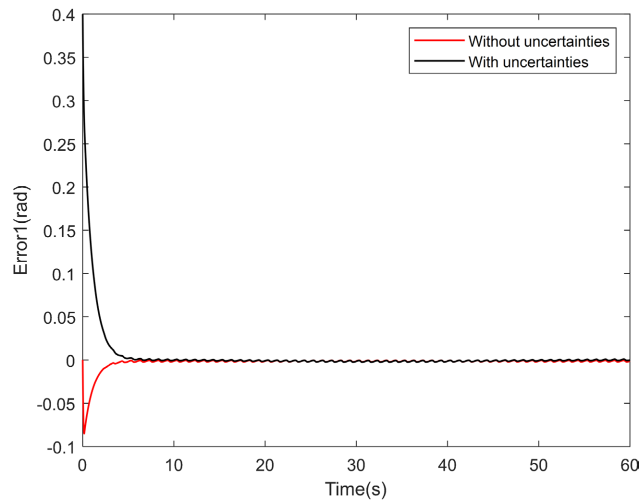

Figure 3. The trajectory tracking error (Error1) of hip angle θ_1 in both conditions.

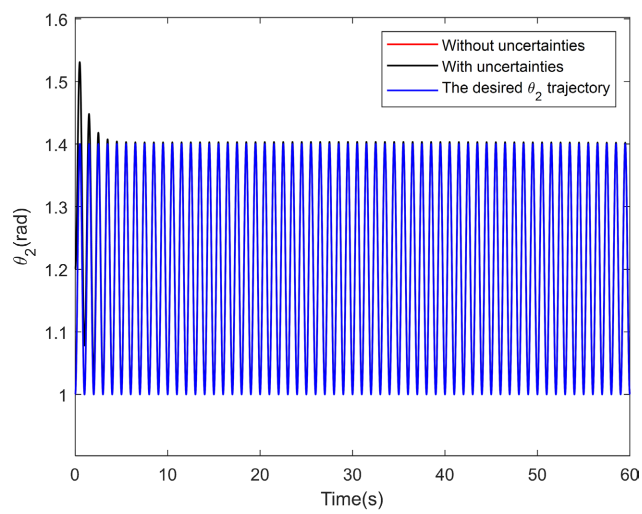

Figure 4. The trajectory tracking performance of knee angle θ_2 in both conditions.

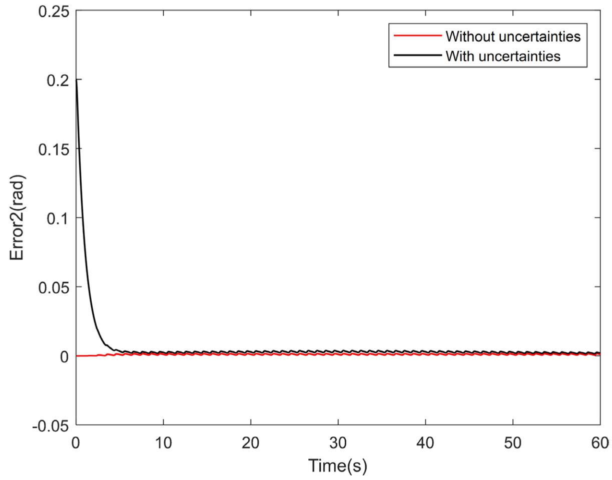

Figure 5. The trajectory tracking error (Error2) of knee angle θ_2 in both conditions.

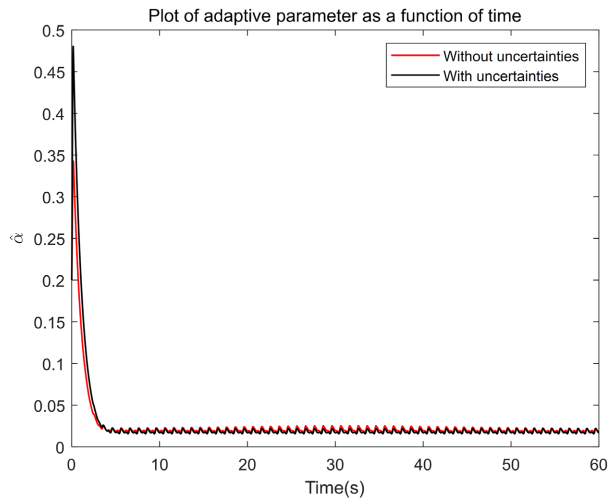

Figure 6. The adaptive parameter $\hat{\alpha}$ in both conditions.

5. Conclusion

This paper designs an adaptive robust control method with leakage-type control law for the trajectory tracking issue of exoskeleton robots. We formulate the close-loop dynamic model of the constrained mechanical system, building a connection between the unconstrained dynamic model and constraints in the real world. In addition, we consider the uncertainties in the system which are unknown but bounded. Then we design an adaptive robust control method based on leakage-type control law. Finally, we conduct the numerical simulations in both conditions (with or without uncertainties) of the exoskeleton system. The results demonstrate that our proposed method has good trajectory tracking performance and robustness. Our work is crucial for the dynamic modelling of nonlinear systems and for improving the accuracy and robustness of control systems. In future work, we will establish dynamic models that are more in line with real-world constraints and strive to conduct experimental verification. Furthermore, the optimal identification of control parameters is also one of our plans.

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Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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