

# Two Letters on Music of Giovanni Battista Benedetti to Cipriano de Rore. Part II: Transcription and Translation into English

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## Abstract

In two letters written to the famous musician Cipriano de Rore, Giovanni Battista Benedetti discussed topics related to musical theory. The letters were published in the *Diversarum speculationum mathematicarum et physicarum liber* of 1585. Despite their great relevance, though, no modern translation of the letters from the original Latin into a contemporary language in full has been attempted. This paper intends to fill this gap by presenting a semidiplomatic transcription and a faithful translation into English.

## Keywords

Theory of Music, History of Acoustic, Vibrations of Strings, Migration of Pitches, Musical Proportion Theory

## 1. Transcription

The transcription comes from Benedetti's *Diversarum speculationum mathematicarum & physicarum liber* of 1585, printed by Bevilaqua in Turin; first letter pp. 277-278, second letter pp. 279-283 (Benedetti, 1585).

IO. BAPTISTAE

BENEDICTI

Patritij Veneti Philosophi

Diversarum Speculationum

*Mathematicarum & Physicarum*

Liber.

Taurini, Apud Haeredem Nicolai Bevilaque, MDLXXXV.

(Superioribus permissum.)

*TRACTATUS QUI IN HOC  
volumine continentur.*

Theoremata Arithmetica  
De rationibus operationum perspectivae  
De Mechanicis  
Disputationes de quibusdam placitis Aristotelis  
In quintum Euclidis librum  
Physica et Mathematica  
Responsa per Epistolas.

The copy consulted belongs to the library Guido Castelnuovo, of Sapienza University of Rome.

The following is a semi-diplomatic transcription of the Latin text (we chose this type of transcription because it allows a fairly clear idea of the original text while maintaining an easy readability). The shortenings are enclosed in square brackets. According to the modern transcription of the Latin text, accents are omitted, u and v are interchanged where necessary, and figures are left in their original form (duplicate figures are however omitted). Our transcription closely follows the original Latin text, differing only slightly from the version in [Reiss \(1924\)](#), which includes more interpretative editing that may obscure the text's original structure and features. Typos are correct, but signaled. The bold numbers in square brackets refer to the end of pages of the original text.

### 1.1. First Letter

DE INTERVALLIS MUSICIS  
*Cypriano Rore Musico celeberrimo.*

Opinio Hectoris Eusonij Cypriane mi dilectissime, vera non est, quod aliquis recte possit intelligere rationes consonantiarum musicae, absque cognitione illarum mediante ipso sensu, imo nemo po[tes]t calere theoriam musices, nisi aliquo mo[do] versatus sit in praxi. Qu[omod]o enim cognosci poterunt quid nam sint diapason, diapente, diatesseron, ditonus, semiditonus, hexacordum maius, aut minus, & consonantiae ex ijs cum diapason compositae, absque earum praxi? unde sequetur neq[ue] etiam cognosci posse intervalla dissonantia. Et purus practicus non intelliget quid sit octava, quinta, quarta, tertia maior, tertia minor, sexta maior, sexta minor, decima maior, decima minor, undecima, duodecima, decimatertia maior, aut minor, aut decimaquinta, & aliae, ita ut ad comparandam perfectionem musicae necessarium sit, & theoriam & praxim addiscere. Cum pr[a]eterea Ludovicus Folianus aperte monstrarit (etiam si id a diatonico sintono Ptolomei desumpserit) reperiri duos tonos, maiorem, & minorem, idest sesquioc-tavum, & sesquinonum, & tria semitonia, maius, minus, & minimum, idest sesquiquintumdecimum, qui est maius, sesquivigesimumquartum idest minimum, & mediocre, ut .27. ad .25. quae proportio superbipartiens vigesimasquintas appellatur, & cum cognoverit semiditonus consonantem esse sesquiquintum, ditonus sesquiquartum, & hexacordum minus, ut .8. ad .5. quae proportio dicitur

supertriptiens quintas, & hexachordum maius, ut .5. ad .3. haec autem vocatur super-bipartienstertias; omnium simplicium consonantiarum cognitioni, extremam imposuit manum. Et quia tibi etiam ostendere promisi in modulationibus [277] haec omnia intervalla servari, ideo ad te mitto septem hic subscripta exempla, in quorum primo, & secundo, inter diesim [in the English text is diesin], et .b. in superiori, agnosces intervallum minimi semitonij, & si ibi sit diesis, tanquam terminus ad quem, et .b. tanquam terminus a quo: quod autem inter diesim [in the Latin text is diesin] et .b. sit semitonium minimum facile agnosces si subtraxeris decima[m] minore[m] a maiori, quam facit superius cu[m] inferiori, idest cu[m] bassu.

Qua quidem, modulatione tu etiam usus es in cantilena illa, quae Galica lingua incipit. Hellas comment. Eadem, ego quoque in meis cantilenis latino sermone compositis, quae Moteta [in the Latin text is Moreta] vocantur aliquando usus sum.

Sed in tertio exemplo invenies semitonium maius, necessario genitum in superiori, si sextam maiorem cum bassu efficere volueris, quia tenor a ditono cum superiori ad diapentem, & ad unisonum cum bassu procedit ubi quiescit, progrediendo postea bassus ad semiditonus cum tenore, tunc si a proportione huius septimae, quae est ut .9. ad .5. hoc est superquadripartiensquintas demptum fuerit hexachordum maius, seu sexta maior, quae est ut .5. ad .3. remanebit proportio .27. ad .25. quae maior est quam .32. ad .30.

In quarto exe[m]plo habebis semitonium minus in superiori, quod quidem remanet ex subtractione ditoni co[n]sona[n]tis ab diatessaron co[m]prehensa a superiori cum tenore.

In quinto exemplo videbis tonum minorem, & tonum maiorem successive unum post alium in tenore, detrahendo primo semiditonus a diatessaron, quod superius facit cum tenore, vel detrahendo diapente ab hexachordo maiori, quod facit tenor cum bassu, unde remanet tonus minor sesquionus, detrahendo postea diatessaron a diapente, quod superius facit cum tenore, remanebit tonus maior sesquioctavus. In sexto exemplo deinde videbis tenorem ascendere per duos tonos minores successive unum post alium in tenore, si de[m]pseris semiditonus a diatessaron cu[m] superiori.

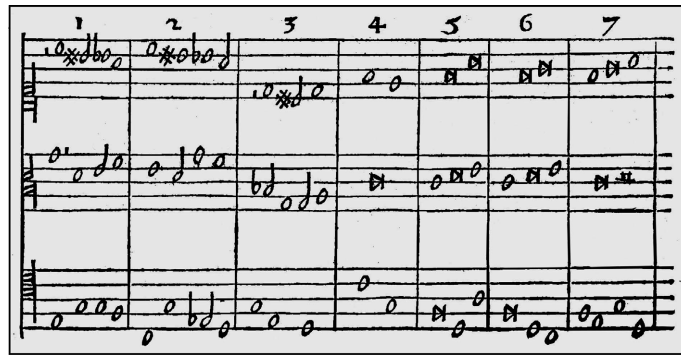
In .7. exemplo demum videbis superiorem ascendere per duos tonos maiores successive unu[m] post aliu[m] si dempseris diatessaron a diapente, quod facit tenor cu[m] superiori. [278].

## 1.2. Second Letter

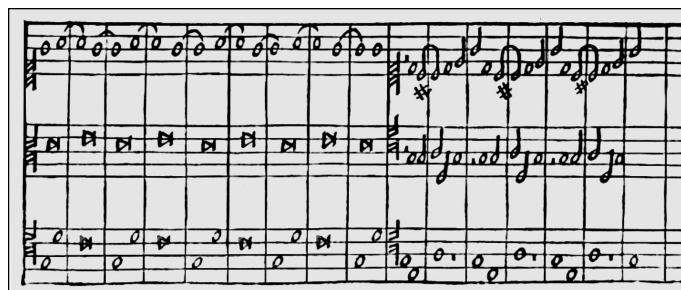
De eodem subiecto

AD EUNDEM.

Quod alias tibi dixi, verum est, quod necessarium nullo modo sit, ut modulando, desinat cantilena in eodem tono (quod Graeci phthongum appellant) a quo inc[a]epit immo necessario semper fere, altius aut depraessius terminatur, per differentiam alicuius intervalli aequalis, vel multiplicis ipsi commati sesquioctavesimae, quod



quidem comma, quamvis cantabile non sit, insensibiliter tamen generatur, & toties ab aliqua parte ipsius cantilenae posset dictum comma generari, versus acutum, vel grave, quod in fine ipsius cantilenae, vocis phthongus reperiatur distans a primo per intervallum alicuius toni sesquinoni, seu sesquioctavi plus, minus[que], ut in subscripto exemplo clare videre potes in prima figura, ubi superius a .g. primae cellulae ad .g. secundae, interest unum comma, eo quod progrediens superius in prima cellula ipsius cantilenae a quarta ad quintam cum tenore, ascendit per tonum sesquioctavum, a prima cellula deinde ad secundam, tenor ascendit similiter per tonum sesquioctavum cum transeat a quinta ad quartam, quod facit cum superiori, in secunda cellula postea, cum superius descendat a maiori sexta ad quintam, quod facit cum bassu, seu a quarta ad tertiam minorem, quod facit cum tenore, tunc descendit per tonum sesquinonum, ita quod non revertitur ad eundem phthogum, ubi prius erat in prima cellula, sed reperitur per unum comma altius, quod quidem comma est differentia inter tonum sesquioctavum & sesquinonum, ut alias tibi demonstravi.



Progrediendo igitur hoc modo, videbis quod cum tenor a secunda cellula ad tertiam transeat a tertia minori ad quartam, quod facit cum superiori, descendit per tonum sesquinonum, unde in tertia cellula altius remanet quam in prima per unum comma, in qua tertia cellula, cum iterum transeat superius a quarta ad quintam, quod facit cum tenore, elevatur per tonum sesquioctavum, proseguendo deinde tali ordine, videbis in quarta cellula cantilenam auctam per duo commata, in sexta, autem cellula per tria commata, in octava vero per .4. commata, unde hac methodo, si cantilena prolixior debito esset, vel si talia intervalla frequentiora reperirentur, posset cantilena a principio ad finem differre per .9.

commata, & plus etiam, quae quidem [279] intervalla superant tonum sesquionum, & si essent .10. commata superarent tonum sesquioctavum, eo quod aggregatum ex .9. commatibus continetur sub istis duobus terminis hoc est .150094635296999121. et .134217728000000000. quae quidem proportio maior est proportione sesquinona, summa vero .10. commatum continetur sub. 12157665459056928801. et .10737418240000000000. quae proportio maior est tono sesquioctavo, quod autem dico de ascensu cantilenae, idem assero de eiusdem descensu, & hoc non tantum per intervallum illius commatis, quod est differentia toni maioris a minori, sed etiam per illud quod est differentia semitonij maioris a minori, ut in secundo exemplo hic subscripto videre est in descensu cantilenae per comma & comma, ut differentia inter semitonia maiore & minore, ubi in prima cellula discedens bassus a quinta cum superiori, & ab unisono cum tenore descendens ad tertiam minorem cum ipso tenore, facit cum superiori septima[m] maiorem, quae est ut .9. ad .5. superquadripartiense quintas scilicet, a qua discedens postea superius, ut faciat cum bassu sextam maiorem, descendit per semitonium maius, a qua sexta maiori descendens bassus, & ascendens per quartam, efficit cum dicto superiori tertia[m] maiorem, a qua discedens superius, ut efficiat quartam cum ipso bassu (qui quidem bassus transit in tenorem) ascendit per semitonium minus, differens a semitonio maiori per unum comma, unde cantilena remanet depressa per unum comma, cum deinde idem faciat inter tertiam, & quartam cellulam, per aliud comma descendit, & sic toties facere posset, ut postremo valde deprimatur cantilena a primo phthongo.

Quod autem hic supradictum est, circa instrumenta artificialia non accidit, quia propter organa, & clavicimbula concordantur certo quodam ordine, ita ut omnes consonantiae, excepta diapason, seu octava, sint imperfectae hoc est, aut diminut[a]e, aut superantes a iusto, ut exempli gratia, omnes quintae sunt diminutae, quartae vero sunt excessivae, quod quidem sit, ut tertiae, & sextae, non multum auribus dissonent, eo quod si quintae omnes, & quartae, perfectae essent, tunc omnes sext[a]e, & tertiae intollerabiles essent, & a perfectis differrent per unum comma, quod manifestum nobis erit hoc modo, accipiamus tres diapentes, seu quintas, consequenter successivas unam post aliam, hoc est tres proportionales sesquialteras, quarum aggregatum erit ut .27. ad .8. quae proportio, dicitur tripla supertripartiense octavas, & quae a practicis [280] appellaretur tertiadecima maior, ut exempli gratia, esset gamaut cum secundo elami, tunc talis tertiadecima valde odiosa esset sensui auditus, a qua, si dempta fuerit diapason, seu octava, remaneret quoddam hexachordum maius, seu sexta maior, auribus valde inimica, sub proportione .13. ad .8. sed haec proportio differret a proportione superbipartientetertias, p[er]fecti hexachordi maioris, hoc est sextae maioris consonantis, per proportionem sesquioctuagesimam, hoc est per unum comma, quod quidem est etiam differentia aggregati trium sesquialtearum, a tertiadecima maiori consonanti, hoc est excessus proportionis triplae supertripartientis octavas, supra triplam sesquiterciam, quae est summa ipsius duplae cum superbipartiente tertias.

A tali summa igitur trium sesquialteraarum efficitur tertiadecima maior dissonans excedens consonantem per unum comma (cuius proportio est .81. ad .80.) quae consonans continetur in proportione .10. ad .3. ut supra dixi.

Haec igitur est vera ratio, propter quam debemus comma distribuere in organis & clavicymbalis, cu[m] ab aggregato trium quintarum producat talis excessus supra perfectam, seu consonantem tertiamdecimam maiorem, quod quidem aggregatu[m], cum demptum fuerit a quintadecima, relinquet nobis tertiam minorem dissonantem, & mancam per eundem excessum a consonanti quae quidem tertia minor dissonans subtracta a diapente seu quinta perfecta, relinquet nobis tertiam maiorem dissonantem, quae consonantem excedit per eundem excessum commatis, & haec demum tertia maior dissonans, dempta ex diapason, seu octava, relinquet nobis hexachordum minus, hoc est sextam minorem dissonantem, & mutilam a consonanti per eundem excessum commatis. Vide ergo, quod cum ex aggregato trium quintarum, generetur tale intervallum, necessarium sit illud dividere per tres partes aequales, deinde ipsas distribuere, ut recte docet Doctissimus Zarlino, in libris de institutionibus musicae. Quamvis nec ipse, nec alius (quod sciam,) ullam fecerit mentionem de huiusmodi commatis generatione, ut supra dixi, quod est differentia iam dicta, divisibilis per tres partes aequales distributivas, quae a tribus quintis perfectis generatur, sive componitur, hoc est a tribus excessibus trium quintarum perfectarum supra tres quintas utiles in ipse dictis instrumentis.

Sed quia sensus auditus non potest exacte cognoscere debitam quantitatem excessus, vel defectus, intendendo vel remittendo chordas instrumentorum, ideo ha[n]c viam sequutus sum.

Sit exempli gratia, hic subscriptus ordo lignorum tangentium seu pinarum incipiens ab .G. desinens ad .g. ita quod inter ipsos terminos sit ea consonantia quae vocatur vigesimasecunda, quaero primum .b. inter .D.E. quod est nigrum ipsius Elami gravissimum, quod grosso modo facio consonans cum .G. gravissimum per sextam minorem, deinde cum ipso primo .b. ipsius Elami co[n]chordo [in the latin text it is concordo] suum octavum & quintumdecimum, quo perfectius possum; deinde accipio .b. molle secundum ipsius bfabmi quod co[n]chordo cum .b. primo ipsius Elami per quintam imperfectam, deinde cum hoc .b. secundo ipsius bfabmi co[n]chordo secundum .f. per quintam similiter imperfectam, cum quo .f. postea co[n]chordo tertium .c. persimilem quintam quem tertium .c. postea conifero cum secundo .b. ipsius elami, ita quod inter se consonent per sextam maiorem tolerabilem, & si sic invenio, tunc nihil muto has tres chordas hoc<sup>1</sup>

G A b B C \* D b E F \* g \* a b b c \* d b e f \* g \* a b b c \* d b e f \* g \*  
1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 4 4

[281] est .b. secundum ipsius bfabmi, .f. secundum, et .c. tertium, sed si dictum tertium .c. valde dissonans esse cum .b. secundo ipsius elami, tunc ipsum .c. intendendo, aut remitto, quo usque aliquo modo sit consonans per sextam maiorem

<sup>1</sup>In the original Latin text in the table below, *b* between *D* and *E*, is wrongly missed, analogously the order *GABb* should be changed in *GAbB*.

aliquantulum excessivam cum .b. secundo ipsius elami, cum quo postea .c. consonare aliquantulum facio .f. secundum per quintam defectivam, & cum hoc demum .b. secundum ipsius. bfabmi, quo facto concordo secundum .c. cum tertio per octavam, cum quo secundo .c. postea concordo tertium .g. per talem quintam, quod ipsum tertium .g. cum secundo .b. ipsius bfabmi consonet tolerabiliter per sextam maiorem aliqua[n]tulum excessivam deinde cum isto tertio .g. concordo tertium .d. per talem quintam, ita quod ipsum .3.d. concordet tolerabiliter cum .2.f. per sextam maiorem excessivam, postea cum hoc .3.d. concordo .2.d. per octavam perfecte cum quo .2.d. postea concordo .3.a. per quinta[m], ut in alijs factum est, ita ut cu[m] .2.c. co[n]sonet talis sexta maior, ut supra dictum est, cum quo .3.a. postea concordo .3.e. per quintam, ut dictum est, ita quod cum .3.g. faciat sextam maiorem, ut supra, postea cum hoc .e. concordo .2.e. per octavam cum quo concordo .b. quadrum tertium per quintam, ut dictum est, ita quod cu[m] .2.d. faciat sextam maiorem similem alijs superius dictis, cum quo .b. quadrato tertio concordo tertium nigrum ipsius .f. per quintam, ita quod cum .3.a. faciat sextam maiorem, ut supra, deinde cum hoc concordo .2.f. nigrum per octavam, cum quo, per quintam concordo 3.c. nigrum ita quod cum .2.e. faciat sextam dictam, demum cu[m] hoc concordo .4.g. nigrum per quintam, ita quod faciat cum .3.b. quadrato sextam dictam, & sic ad ultimam quintam pervenio, supra quod .g. nigrum nulla quinta amplius reperitur, postea cum istis chordis concordo per ottavas omnes alias ab acutis ad graves.

Valde etiam admiratione dignum est, quod perfectiores quaeque consonantiae, ita in harmonica divisione sibi invicem convenient, ut diapason cum diapente, cum diapasondiapente, cum ditono, cum hexachordo maiori cum bisdiapason, cu[m] decimaseptima maiori. Nam in ipsa diapason, harmonice locatur diapente in parte graviori, & diatessaron in acutiori. In diapente vero harmonice locantur tonus maior in parte graviori, & tonus minor in acutiori. In ditono harmonice locantur ditonus in parte graviori, & semiditonus in acutiori. In hexachordo maiori, harmonice locantur diatessaron in parte graviori, & ditonus in acutiori. In diapasondiapente, harmonice locantur diapason in parte graviori, & diapente in acutiori. In bisdiapason, harmonice locantur decima maior in parte graviori & hexachordum minus in acutiori. In decimaseptima maiori harmonice locantur diapason diapente in parte graviori, & hexachordum maius in parte acutiori. Ita quod tonus sesquioctavus in ditono, proportionalis est ipsi ditono in diapente. Tonus vero sesquiquonius in ipso ditono, proportionalis est triemitonio, vel sesquintonio seu semiditono (quod idem est) in diapente. Ditonus autem in diapente; proportionalis est ipsi diapente in diapason. Sesquintonus vero in diapente, proportionalis est diatessaron in diapason. Et sic de singulis. Ita quod tonus sesquioctavus in ditono, ditonus in diapente, diatessaron in hexachordo maiori, diapente in diapason, diapason in diapason diapente, decima maior in bisdiapason, diapasondiapente in decima septima [282] maiori, omnia sibi invicem sunt proportionalia, idem etiam dico de reliquis partibus, cum relatae fuerint ad sua tota.

Nec alienum mihi videtur a proposito instituto, speculari modum generationis

ipsarum simplicium consonantiaru[m]; qui quidem modus fit ex quadam aequatione percussionum, seu aequali concursu undarum aeris, vel conterminatione earum. Nam, nulli dubium est, quin unisonus sit prima principalis auditu[ue]; amicissima, nec non magis propria consonantia; & si intelligatur, ut punctus in linea, vel unitas in numero, quam immediate sequitur diapason, ei simillima, post hanc vero diapente, caeteraeq[ue]. Videamus igitur ordinem concursus percussionum terminorum, seu undarum aeris, unde sonus generatur.

Concipiatur igitur mente monochordus, hoc est chorda distenta, quae cum divisa fuerit in duas aequales partes a ponticulo, tunc unaquaeq[ue]; pars eundem sonum proferet, & ambae formabunt unisonum, quia eodem tempore, tot percussiones in aere faciet una partium illius chordae, quot & altera; ita ut undae aeris simul eant, & aequaliter concurrant, absque ulla interfectione, vel fractione illarum invicem.

Sed cum ponticulus ita diviserit chordam, ut relicta sit eius tertia pars ab uno latere, ab alio vero, du[a]e terti[a]e, tunc maior pars, dupla erit minori, & sonabu[n]t ipsam diapason consonantiam, percussiones vero terminorum ipsius, tali proportione se invicem habebunt, ut in qualibet secunda percussione minoris portionis ipsius chordae, maior percutiet, seu concurret cum minori, eodem temporis instanti, cum nemo sit qui nesciat, quod quo longior est chorda, etiam tardius moveatur, quare cum longior dupla sit breviori, & eiusdem intensionis tam una quam altera, tunc eo tempore, quo longior unum intervallum tremoris perfecit, brevior duo intervalla conficiet.

Cum autem ponticulus ita diviserit chordam, ut ab uno latere relinquuntur duae quintae partes, ab alio vero tres quintae, ex quibus partibus generatur consonantia diapente; tunc clare patet, quod eadem proportione tardius erit unum intervallum tremoris maioris portionis, uno intervallo tremoris minoris portionis, quam maior portio habet ad minorem; hoc est tempus maioris intervalli ad tempus minoris erit sesquialterau[m]; quare non co[n]venient simul, nisi perfectis tribus intervallis minoris portionis, & duobus maioris; ita quod eadem proportio erit numeri intervallorum minoris portionis ad intervalla maioris, quae longitudinis maioris portionis ad longitudinem minoris; unde productum numeri portionis minoris ipsius chordae in numerum intervallorum motus ipsius portionis, aequale erit producto numeri portionis maioris in numerum intervallorum ipsius maioris portionis; quae quidem producta ita se habebunt, ut in diapason, sit binarius numerus; in diapente vero senarius; in diatessaron duodenarius, in hexachordo maiori quindenarius; in ditono vicenarius, in semiditono tricenarius, demum in hexachordo minori quadragenarius: qui, quidem numeri non absque mirabili analogia conveniunt invicem.

Voluptas autem, quam auditui afferunt consonantiae fit, quia leniuntur sensus, quemadmodum co[n]tra, dolor qui a dissonantijs oritur, ab asperitate nascitur, id quod facile videre poteris cum conchordantur organorum fistulae. [283]

## 2. Translation into English

The translation from Latin to English has been made quite close to the original

text, even at the expense of elegance of exposition; punctuation has often been changed and not always the verb tenses are respected. The following major caveats apply.

**Intervals.** Benedetti uses the Greek terminology of intervals, in which the most important consonances are the diapason (2:1), the diapente (3:2), and the diatesaron (4:3). However, this nomenclature is often supplemented or replaced by the Latin terminology based on the division of the octave into seven intervals (octave, fifth, fourth).

For the elemental components of the musical scale, Benedetti uses the language of the time, which is established and coincides with the modern language for whole tones, major (9:8) and minor (10:9), but differs for the semitones, for which there is still no established use and for which even Benedetti is sometimes not precise in his definition. He introduces three kinds of semitones, which he calls *minimum*, *minor* and *maius*, characterized by the ratios (25:24), (16:15) and (27:25) respectively. Following Fogliano, who in the *Musica theorica* defined: major [maius] interval as the ratio 27:25, minor [minus] 16:15 and minimum [minimum] 25:24, which are in descending order [6, ff. 18v-19r, and f. 31v], on its part Zarlino called (16:15) and (25:24) *maggiore* and *minore* in the *Dimostrazioni harmoniche* and in the *Sopplimenti musicali* (Zarlino, 1588) gave three kinds of semitone.

We have chosen not to translate Benedetti's Latin names of the semitones literally, so as not to introduce ambiguity. For example, Benedetti's semitone minor is defined by the same ratio (16:15) as the modern major semitone. Therefore, we have translated minimum with *minimal*, minor with *small* and maius with *maximal*.

**Ratios.** Ratios (referred to by the then-common term proportions) defining musical intervals are divided into superparticulars and superpartients, as used from the Middle Ages, following the treatise of Nicomachus (1926); this nomenclature was also adopted by Boethius. In medieval Latin musical literature, the superparticular genus gives rise to species defined by the prefix sesqui, a Latin term originally denoting a ratio of (3:2), but generalized to cover intervals of the kind  $(n+1)/n$ . Since there is no simple consolidated English nomenclature, we insist on using the term sesqui- $n$ th for a superparticular interval of order  $n$ . In this, we follow Zarlino example in his translation from Latin to Italian (Zarlino, 1562: pp. 21-38). For example,  $n=10$  corresponds to the ratio (11:10), which can be called a sesqui-tenth ratio. We have made exceptions for  $n=2,3,8,9$ , for which there are quite common English names, corresponding respectively to sesquialtera, sesquitercian, sesquioctave, and sesquinona.

The superpartient genus refers to ratios of the form  $(n+a)/n$  where  $a>1$ , so as not to fall into the superparticular genus. It gives rise to ratios that are called super- $a$  partient  $n$ th. For  $a=2$  and  $n=25$ , we have for example the super-bi-partient twenty fifth (27:25), the ratio of the maximum semitone. A ratio defined by  $(kn+p)/n$  is named  $k$ -th super  $p$  partient  $n$ -th. In the present case, with

$k = 3$ ,  $p = 3$ ,  $n = 8$ , we have 27:8, and the ratio is named super-three-partient-octave.

## 2.1. First Letter

### The Intervals of Music

To Cipriano de Rore, a renowned musician.

My most beloved Cipriano, the opinion of Hector Eusonius<sup>2</sup>, according to which one can understand the meaning of consonances without knowing them through the senses, is wrong, and no one can be an expert in music theory unless he has some practical experience<sup>3</sup>. And how can one be aware [Latin: cognosci] of the diapason [octave], the diapente [fifth], the diatessaron [fourth], the ditone [major third], the semitone [minor third], the hexachord [sixth] major and minor, and the consonances derived from them composed with the diapason, without their practice? From which it follows that he will not even be able to know the dissonant intervals.

But the ordinary practitioner cannot understand [Latin: addiscere]<sup>4</sup> what the octave, fifth, fourth, major third, minor third, major sixth, minor sixth, major tenth, minor tenth, eleventh, major or minor twelfth or others are<sup>5</sup>. Therefore, to achieve perfection in music it is necessary to learn both theory and practice. Furthermore, as Ludovico Fogliano (1529) clearly showed<sup>6</sup> (although he derived it from Ptolemy's diatonic syntonic system), there are two whole tones, major and minor, given by the sesquioctave [9:8] and the sesquinone [10:9], and the three semitones, small semitone [minus], minimal semitone [minimum] and maximal semitone [maius]<sup>7</sup>, given by sesqui-fifteenth [16:15], which is the small, sesqui-twenty fourth [25:24] which is the minimal, and the maximal, [Latin: mediocre] which is like 27 to 25, which is called the super-bipartient twenty fifth proportion,<sup>8</sup> and as you know the consonant semitone is the sesqui-fifth [6:5], and the ditone [is] the sesqui-forth [5:4], the minor hexachord [is] as 8 to 5, whose proportion is

<sup>2</sup>Ettore Ausonio (1520?-1570). Mathematician at the Savoy court before Benedetti (Frank, 2014).

<sup>3</sup>Here practice may be given the meaning used by musicians that is *Musica practica*, or it may be assumed to refer to an empirical approach since Benedetti considered music to be a mixed mathematics based on experience and not on metaphysics, as is the case in the Pythagorean tradition. From the rest of the letter it could be deduced that Benedetti was inspired in his view by Lodovico Fogliano (1475-1542); see ([6, f. 14r]). But it could also be that his conclusion was derived from the newly discovered Greek texts on music theory.

<sup>4</sup>The combination of theory and practice is not strange to Benedetti, who was a mathematician, and considered mathematics to be the highest form of knowledge, and in this perspective he cannot fail to give importance to theoretical knowledge which for him coincides with mathematical knowledge.

<sup>5</sup>Here Benedetti uses the modern denomination, partly repeating the intervals mentioned above.

<sup>6</sup>Benedetti is referring to Fogliano's *Musica theorica* of 1529.

<sup>7</sup>In the Latin text, the order is maius, minus, and mininum, but this should be a typo, because otherwise there is no consistency with the following.

<sup>8</sup>The interval (16:15) is sometimes referred to in the Latin text as a major semitone, sometimes as a minor semitone (the latter being the more common acceptance). In the English translation, we have called it a small semitone, everywhere. In Duffin (2017: p. 242) a special emphasis is given to the Latin term *mediocre* which is assigned the value of 135:128, intermediate between 16:15 and 25:24, an interval not found in Fogliano, to which Benedetti seems to refer. We more simply assume that this was if not a typo, an imprecise wording to mean maximal.

called super-threepartient fifth and the major hexachord as 5 to 3, which is also called the third super-bipartient third; and [Fogliano] reached the definitive knowledge of all the simplest consonances. And here I will show you as promised the modulation of all intervals, [277] so I attach these following seven examples (see **Figure 1**).<sup>9</sup> In the first and second of [these examples] between  $\sharp$  and  $\flat$ .<sup>10</sup> we recognize in the superius the interval of the minimal semitone. If you take the [natural] B as the ad quem term and the B flat as the a quo term, then between natural B and B flat there is a minimal semitone. You can easily recognize this if you subtract the minor tenth from the major tenth, that the superius makes with the inferior, that is, with the bassus.<sup>11</sup> This modulation was also used by you in the chant that in French language begins with *Hellas comment*.<sup>12</sup> I, too, have sometimes used it in the chants composed in Latin called motets.<sup>13</sup>

**Figure 1.** Modern transcription of Benedetti's examples of the first letter. From bottom to top in the staff: bassus, tenor, superius.

But in the third example a maximal semitone, generated in the upper part,

<sup>9</sup>In the figure, the vertical lines do not delimit the bars of a single piece, but six independent short pieces of music. The voices according to Benedetti are bassus, tenor, superius.

<sup>10</sup>In modern notation, in the order, B natural and B $\flat$ , where Benedetti's sharp coincides with B.

<sup>11</sup>Benedetti assumes that the two tenths in the first example are consonants, and in particular that they coincide with the major tenth between the notes G and B and the minor tenth between the notes G and B $\flat$ . The difference between the two tenths is equal to the difference between the major and minor thirds, given by  $5/4:6/5 = 25/24$  which is exactly the semitone called minimum by Benedetti (and Fogliano).

<sup>12</sup>Second part of the last madrigal of De Rore's madrigal collection (*De Rore*, 1569).

<sup>13</sup>Benedetti implicitly claims to be proficient enough in music theory to compose motets in Latin himself.

necessarily intervenes if one wants to produce a major sixth [5:3] with the bassus because the tenor goes from the ditone to the diapente with the superius, in unison with the bassus where it rests, advancing the bassus to a semiditone [a minor third] toward the tenor. Therefore, if from the proportion of this seventh, which is 9 to 5, called super-forth partient fifth, it will be subtracted the major hexachord, which is as 5 to 3, we are left with the proportion 27 to 25<sup>14</sup> which is greater than 32 to 30.<sup>15</sup>

In the fourth example we will have the small semitone [16:15] in the superius which is what remains after subtracting the consonant ditone [major third 5: 4] from the diatessaron between the superius and the tenor.<sup>16</sup>

In the fifth example we will see a minor whole tone and a major whole tone following one after the other in the tenor, first by subtracting the semiditone (6: 5) from the diatessaron, that the superius makes with the tenor, or subtracting the diapente from the major hexachord (5:3), that the tenor makes with the bassus, giving the minor tone sesquionona [10:9]. Subtracting the diatessaron from the diapente that the superius makes with the tenor leaves the major sesquioctave.<sup>17</sup>

In the sixth example you will see the tenor rising by two minor whole tones in succession, if the minor third is subtracted from the fourth against the superius.<sup>18</sup>

Finally, in the seventh example, you will see that the superius raises two major whole tones in succession, by subtracting the diatessaron from the diapente, which the tenor makes with the superius [278].<sup>19</sup>

## 2.2. Second Letter

On the same subject.

To the same.

What I have told you elsewhere is true that it is not at all necessary for the singing of a chant to end on the same note (which the Greeks call phthongo)<sup>20</sup> with which it began. In fact, it is almost always the case that it ends either higher or lower, with the difference of some equal or multiple interval of the sesqui-eightieth

<sup>14</sup>Note that here Benedetti rightly defines the interval (27:25) as a maximal semitone, correcting the oversight outlined above.

<sup>15</sup>That is (16:15) which is the small semitone.

<sup>16</sup>The difference between the fourth and the major third is given by  $4/3:5/4 = 16/15$ , which is exactly the small semitone.

<sup>17</sup>The fourth and fifth are separated by a (9:8) tone.

<sup>18</sup>The minor third subtracted from the fourth gives  $4/3:6/5 = 10/9$ .

<sup>19</sup>The following table summarizes the value of semitones noticed by Benedetti in this first letter; the first line is for Benedetti's names, the second line for modern names and the third line for the sizes of the semitones.

Major tone	minor tone	minimal semitone	small semitone	maximal semitone
Major tone	minor tone	minor semitone	major semitone	maximum semitone
(9:8)	(10:9)	(25:24)	(16:15)	(27:25)

<sup>20</sup>It is actually a Latin term, phthongus, of Greek origin which means sound, note; in the following we will render it as *pitch*.

comma [81:80], which, although it is not singable, is nevertheless generated imperceptibly. And often, somewhere in the chant, this comma can be generated, toward the acute or toward the grave and for this reason at the end of the chant the pitch of the voice will be distant from the beginning of an interval approximately or of the sesquinone tone [10:9], or of the sesquioctave one [9:8] above or below, as can be clearly seen in the first figure [column] of the example below (Figure 2), where the superius .g. of the first cell differs by a comma, from .g. of the second cell, because in the first cell the superius of the chant, progressing from the fourth to the fifth with respect to the tenor, rises by a sesquioctave tone.

**Figure 2.** Modern transcription of Benedetti's examples of the second letter. From bottom to top in the staff: bassus, tenor, superius.

The tenor similarly rises from the first cell to the second cell, passing with respect to the superius from a fifth to a fourth, at the end of the second cell, [the tenor] descending from a major sixth [5:3] to a fifth [3:2] with respect to the bassus, but from a fourth [4:3] with respect] to a minor third [6:5] compared to the tenor, thus [the superius] goes down a sesquinona tone [10:9], therefore it does not return to the same pitch it had in the first cell, but is found a comma higher; for a comma [81:80] is the difference between the sesquioctave tone and the sesquinone tone, as I have shown you elsewhere.<sup>21</sup>

Proceeding in the same way, we see how the tenor from the second to the third cell passes from a minor third [6:5] to a fourth [4:3] with respect to the superius, descending by a sesquinona tone, therefore in the third cell the superius remains higher than in first [cell] for one comma; thus in the third cell for the second time the superius passes from a fourth to a fifth, with respect to the tenor, rising by a sesquioctave tone. Then, as this sequence continues, in the fourth cell the chant rises by two comma, in the sixth by three comma, in the octave by four comma; in this way if the chant were to continue, or if such intervals were to be found more frequently, the chant could differ from beginning to end by 9 comma and

<sup>21</sup>By making the difference between a sesquioctave tone (9:8) and a sesquinona tone (10:9), we have  $9/8:10/9 = 81:80$ .

even more, an interval that [279] of course exceeds the sesquinone tone and if 10 comma are exceeded, it exceeds [also] the sesquioctave tone, because the aggregate of nine comma is contained under these two terms:

.150094635296999121..:134217728000000000.<sup>22</sup> which is obviously a larger proportion of the sesquinone tone; 10 comma are contained under

.1215766549056928801..:107374182240000000000 which is a greater proportion of the sesquioctave tone.

**Figure 3.** Modern transcription of Benedetti's examples of the second letter. From bottom to top in the staff: bassus, tenor, superius.

What I said above about the ascent of the chant I also assert about its descent, but this is no longer due to the interval of the comma, as the difference between the major and the minor tone but for that [comma of the same size (81:80)] as the difference between the maximal semitone [27:25] and the small semitone [16:15], as seen in the second example below, for the descent of the chant by comma and comma, that is the difference between the maximal and small semitones. In the first cell (corresponding to cell 10 of **Figure 3**) the bassus descends, from a fifth with the superius and the unison with the tenor, to a minor third with the tenor and makes a major seventh which is as 9 to 5, or super-fourthpartient-fifth. Then the superius descends by a maximal semitone to make a major sixth with the bassus.<sup>23</sup> The bassus, in turn, rises a fourth to make a major third with the superius.<sup>24</sup> The superius then rises from this major third by a small semitone to make a fourth with the bassus [so that the superius] has risen by a small semitone [16:15], which differs by a comma from the maximum semitone; thus the chant remain depressed by a comma.<sup>25</sup>

Then the same happens between the third and fourth cells; it [the chant] descends by another comma and the same thing happens for all [the cells], so that finally the chant is much lowered from the first pitch.

<sup>22</sup>The product of 9 comma, with a modern notation is given by  $(81:80)^9$ , or  $81^9:80^9$ . Making the powers explicit, the numbers reported by Benedetti are obtained. The interval of 9 comma is 1.118 against 1.111 of the minor whole tone, 10/9.

<sup>23</sup>Passing from (9:5) to (5:3) is worth of the interval (25:27), i.e. a maximal semitone.

<sup>24</sup>A major sixth (5:3) subtracted from a fourth (4:3) makes the interval  $5/3:4/3 = 5/4$ , i.e. a major third.

<sup>25</sup>In fact, in the first cell the superius descends by a maximum semitone, while in the second cell it rises by only a small semitone.

But what has been said above does not occur with artfully made instruments. In fact the organs and harpsichords are harmonized [tempered] in some way, so that every consonance except the diapason, or octave, is imperfect, [narrowed] or widened with respect to the just, so that for example all fifths are narrowed, the fourths always widened, so that the thirds and sixths are not too dissonant to the ears, because if all fifths and fourths were just then all the sixths and thirds would be intolerable, and would differ from perfection by a comma, which it is clear in the following. Let us consider three successive diapente or fifths, i.e., three sesquialtera proportions, whose aggregate is 27:8,<sup>26</sup> which proportion we will call triple super-threepartient octave [280] which is called major thirteenth by practitioners, so for example is the gamut with the second Elami.<sup>27</sup> This thirteenth is very unpleasent to the sense of hearing and if a diapason or octave were subtracted from it, it would remain, properly speaking, a major hexachord or a major sixth strongly hostile to the ear with the proportion 13:8; this proportion differs from the super-bipartient third [5:3] proportion of the perfect major hexachord, which is the major sixth consonance, by the sesqui-eightieth proportion, or by a comma, which is also the difference of the aggregate of three sesquialtera [(3:2)<sup>3</sup> = 27:8] from the consonant major thirteenth [10:3],<sup>28</sup> as well as the excess of the triple super-threepartient-octave ratio [27:8]<sup>29</sup> with respect to the triple sesqui-third [10:3]. From the sum of the three sesquialtera is therefore formed the dissonant major thirteenth exceeding the consonance of a comma (whose proportion is 81:80), this consonance is contained in the proportion 10:3 as mentioned above.

This then is the real reason why we must distribute the comma in the organs and in the harpsichords, this [comma] being produced by the excess of the aggregate of the three fifths over the perfect, that is consonant major thirteenth; this aggregate, when subtracted from a fifteenth [two octaves], will leave a dissonant and narrow minor third by the same excess over the consonant; this dissonant minor third, subtracted from a diapente, or perfect fifth, leaves a dissonant major third; which exceeds the consonant for a comma, and the dissonant major third, subtracted from the diapason, or octave, leaves a minor hexachord, which is a dissonant minor sixth, missing from the consonance for a comma. We can see therefore that when this interval is generated by the aggregate of three fifths, it is necessary to divide it into three equal parts and then distribute them as the very learned Zarlino taught in the book *Istitutioni harmoniche*.<sup>30</sup> Although neither [Zarlino] himself, nor others as far as I know, have made mention of this mode of

<sup>26</sup>The aggregate is the sum of three fifths, equal to  $(3:2)^3 = 27:8$ .

<sup>27</sup>The gamut is the lowest note of a scale, in the case of the keyboard proposed by Benedetti just below, it is the lowest G. The second Elami is the note E in the second octave of the same keyboard.

<sup>28</sup>Actually, it is the difference between the aggregate of the three sesquialtera and the major thirteenth and not vice versa.

<sup>29</sup>Curious how Benedetti changed the definition by speaking of the triple super-tripartient instead of the aggregate of 3 fifths; unusual for a mathematician.

<sup>30</sup>Zarlino discussed the presence of a comma in his monochord and his 2/7 of a comma temperament in the *Istitutioni harmoniche* of 1558 (Zarlino, 1562: pp. 123-131). In Zarlino's tempered system the fifth are lowered of 2/7 of a comma while the fourth ascends of the same amount, to maintain the octave. Minor and major third will both be lowered by 1/7 of a comma.

generating the comma, as I said above. This difference, divisible by three equal distributive parts, is generated, or composed, by three perfect fifths, that is, by three excesses of three perfect fifths with respect to three just fifths in the said instrument.<sup>31</sup>

But since the sense of hearing cannot know exactly the right amount of excess, or defect, this method was followed by stretching or releasing the strings of the instruments. For example, let it be that the following order of wooden or keyboard keys starts with  $G_1$  and end with  $G_4$ <sup>32</sup> so that between the terms there is the consonance called twenty second [three octaves].<sup>33</sup> Take the first .b. between DE,<sup>34</sup> which is the lowest black of the Elami [in the following  $E_{1b}$ ],<sup>35</sup> and I roughly make it consonant with the most grave . $G_1$ . for a minor sixth.

Then I concord  $E_{1b}$  with its octave [ $E_{2b}$ ] and fifteenth [ $E_{3b}$ ], as perfectly as I can; and then I take the second .b. of Bfabmi [in the following  $B_{2b}$ ], that I concord with  $E_{1b}$  for a narrow fifth, then I concord  $B_{2b}$  with  $F_2$  for a fifth which is still narrow. From  $F_2$  I then concord  $C_3$  for the analogous [narrow] fifth, I then compare  $C_3$  with  $E_{2b}$ , that [should be] consonant [with it] for a tolerable major sixth, and if that is the case, then I do not change anything in the three notes  $B_{2b}$ ,  $F_2$  and  $C_3$  (Table 1).

**Table 1.** Benedetti's three octave keyboard.

G	A	b	B	C	#	D	b	E	F	#	G	#	A	b	B	C	#	D	b	E	F	#	G	#	A	b	B	C	#	D	b	E	F	#	G	#	
1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	4	4

[281] But if the aforementioned  $C_3$  is too much dissonant with  $E_{2b}$ , then I raise or lower  $C_3$  so that it is a somehow slightly wide consonant with  $E_{2b}$  for a major sixth. Then I make  $C_3$  somewhat consonant with  $F_2$ , for a narrow fifth and finally with  $B_{2b}$ .<sup>36</sup> When that is done, I concord  $C_2$  with  $C_3$  for an octave, and I subsequently concord  $C_2$  with  $G_3$  for a similar [narrow] fifth. This third  $G_3$  is tolerably consonant with  $B_{2b}$  for a major sixth a little widened, then I concord  $G_3$  with  $D_3$  for a [narrow] fifth, which  $D_3$  is concordant with  $F_2$  for a tolerably widened major sixth. Then from  $D_3$  I concord  $D_2$  for a perfect octave, from which  $D_2$  then I concord  $A_3$  for a fifth as done elsewhere so that  $C_2$  is consonant [with it] for a major

<sup>31</sup>Here Benedetti seems to attribute to himself the introduction of the 1/3 of a comma temperament, in which the major sixth and the major thirteenth are just. If this is the case, then Benedetti did not know neither Zarlino's *Dimostrazioni harmoniche* of 1571 (Zarlino, 1571) nor de Salinas' *De musica libri septem* of 1577 (Salinas, 1577), where this temperament is discussed. But it is also possible that he was simply saying that he had found a new way to show the need for 1/3 of a comma temperament.

<sup>32</sup>Here the subscripted numbers in the notes indicate the octave of Benedetti's keyboard of Tab. 1 to which they belong. In the Latin text, flats are represented by *b* and sharps by \*; here we use the modern symbols *b* and #, both of which are considered as black keys.

<sup>33</sup>In the keyboard suggested by Benedetti both flats and sharps are indicated in the text as black keys.

<sup>34</sup>There is no flat between D and E in Benedetti's keyboard as reported in the Latin text; this is a typo which has been corrected here.

<sup>35</sup>In his Latin text, Benedetti uses the Renaissance term Elami for what we now more simply call E (or Mi).

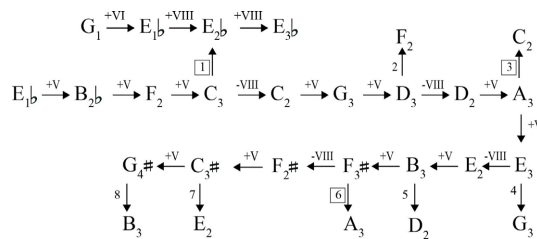
<sup>36</sup>The previous steps serve to check whether the notes  $B_{2b}$ ,  $F_2$  and  $C_3$  are tuned correctly.

sixth, as said above. From  $A_3$  I then concord  $E_3$  for a fifth, as said, so that  $G_3$  makes a major sixth [with  $E_3$ ] as I said above; then from  $E_3$  I concord  $E_2$  for an octave, which I concord with the third .B. square [in the following  $B_3$ ]<sup>37</sup> for a fifth, as I said elsewhere above, so that  $D_2$  makes a major sixth [with  $B_3$ ], as I said elsewhere above; I concord this  $B_3$  with the third black [sharp] of F [in the following  $F_3\sharp$ ] for a fifth so that  $A_3$  makes a major sixth as I said above, so from this I concord  $F_2\sharp$  for one octave from which I concord  $C_3\sharp$  for a fifth so that it makes a sixth with  $E_2$ . Finally from  $C_3\sharp$  I concord  $G_4\sharp$  for a fifth so that it makes a major sixth with  $B_3$ , and so I arrive at the last fifth, since there is no fifth above  $G_4\sharp$ , then from these notes I concord for one octave all the remaining from high to low.<sup>38</sup>

The great perfection the consonances have in their mutual harmonic division,<sup>39</sup> is therefore worthy of admiration, such as the diapason with the diapente [fifth],<sup>40</sup> with the diapasondiapente, with the ditone [major third], with the major hexachord [major sixth], with the bisdiapason, and with the major seventeenth. Thus in the diapason the diapente is placed in the lower part and the diatessaron in the upper part. In the diapente the ditone [major third, 5:4] is harmoniously arranged in the lowest part, the semiditone [minor third, 6:5] in the highest. In the ditone [5:4] the major tone [9:8] in the lowest part and the minor tone in the highest part [10:9] are harmonized. In the major hexachord [5:3] the diatessaron [the fourth] is harmonized in the lowest part, and the ditone [the major third] in the highest part. In the diapasondiapente [3:1] the octave is harmonized in the lowest part and the diapente in the highest. In the bisdiapason, the major tenth is harmoniously arranged in the low part, the minor hexachord in the high part. In the seventeenth major [5:1] the diapasondiapente is located in the lowest part and the major hexachord in the highest part. Just as the sesquioctave of the ditone is

<sup>37</sup>Here and below, square stands for our natural B.

<sup>38</sup>In tuning his keyboard, Benedetti starts from a note  $G_1$ , whose pitch is not specified, and moves forward by 11 narrow fifths and backward and forward by octaves; only the first step, from  $G_1$  to  $E_1b$ , is taken with a minor sixth. In this way he assigns notes to 19 keys (over 35). The remaining keys are assigned a note with forward and backward octaves.



The figure above shows the cycles of fifths, with horizontal arrows. The vertical arrows indicate the control of tuning, looking at previously assigned keys. The Arabic numbers, from 1 to 7, indicate the order in which the control takes place. It is carried out in each cycle of three narrow fifths less one octave, which should form a tolerably widened major sixth; the square box indicates this type of control. There are also intermediate controls, which always look backward for a cycle of three fifths less one octave.

<sup>39</sup>The harmonic division, i.e., the harmonic mean of two numbers  $a$  and  $b$  is the inverse of the mean of the inverse of  $a$  and  $b$ .

<sup>40</sup>In reality, the fifth divides the octave with the arithmetic mean.

proportional<sup>41</sup> to the ditone of the diapente, the sesquinona tone of the ditone is proportional to the triemitone [minor third, 6:5] or sesquitone or semiditone (which is the same), as the semiditone of the diapente. On the other hand, the ditone of the diapente is proportional to the diapente in the diapason. The sesquitone in the diapente is proportional to the diatessaron in the octave. And the same with single intervals. Hence, the sesquioctave tone in the ditone, the ditone in the diapente, the diatessaron in the major hexachord, the diapente in the diapason, the diapason in the diapasondiapente, the major tenth in the bisdiapente, the diapasondiapente in the major seventeenth, [282] all are proportional to each other and I say the same for the remaining parts, when the parts refer to the whole.

Nor is the established argument foreign to me, a way of speculating on the generation of simple consonances, that there really is a certain equality of percussion or an equal concurrence of air waves, or of their termination.<sup>42</sup> In truth, there is no doubt that the unison is the first and principal and most pleasing to hear, the best proper consonance. And so it is understood as the dot and the line or the unity in numbers, followed by the diapason which is very similar to it, after this the diapente and so on.

So let's see the order of the concurrence of the percussion of the boundaries<sup>43</sup> or [seu] of the air waves from which the sound is generated. It is known that when a monochord, a taut string, is divided by a bridge into two equal parts, each part reproduces the same sound and both form the unison because at the same time, as many percussions occur on one part of the string as on the other, so that the air waves arrive together and concur equally without cancelling each other out or breaking each other.<sup>44</sup> \*But when the bridge divides the string so that the third part remains on one side and two thirds on the other, then the major part will be double the minor and [the two parts] will play with the consonance of the diapason. The percussions of the boundaries [percussiones vero terminorum ipsius], of the same [of the string]<sup>45</sup> will have such proportions to each other that at every second percussion of the minor portion of the string, the major portion will strike, or will concord, with, the minor at the same moment. As anyone is aware, the longer the string, the slower it moves.<sup>46</sup> So because the longer is twice the shorter,

<sup>41</sup>It is not clear the meaning of proportional here; may be that instead of "proportional to" one could read "similarly disposed as".

<sup>42</sup>This assumption of equal termination [Latin: conterminatione] of the notes of consonant intervals is not entirely original to Benedetti, it is also partly contained in the Aristotelian *Problemata*, 4.24 (Translated into English by Barker (1989: vol. 2, p. 95) and *De audibilibus* 804a (Barker, 1989) and in Beckman (Cohen, 1984: p. 128).

<sup>43</sup>The exact meaning of the locution "percussiones terminorum" is not fully clear to us. Percussions [percussiones] should refer to the impact of the vibrating string with air (see below); boundaries [termini], could be referred to the surface of the string. Fortunately, here and below, the correctness or not of the translation of the piece does not change the meaning of Benedetti's writing as a whole.

<sup>44</sup>It is clearly stated here that the [periodic] vibration of the string produces [periodic] pulses in the air.

<sup>45</sup>(Palisca, 1994: p. 215), refers ipsius to the octave instead to the string, and translates "percussiones vero terminorum ipsius" as "percussions of the boundary tones of this octave" [the two tones that make the octave].

if both have the same tension,<sup>47</sup> then at the same time the longer makes one interval of vibration the shorter makes two.

When, instead the bridge divides the string so that two fifths remain on one side and three fifths on the other, the consonance of the diapente [fifth 3:2] is generated in these parts, as evident. That is to say, a vibration interval<sup>48</sup> of the larger part will be in the same proportion to a vibration interval of the smaller part [of the longer to the shorter length], i.e. the time [duration] of the interval [of the longer string] to the time of the shorter will be sesquialtera, for which there will be no concordance at the same moment except for three perfect intervals [of time, periods] of the smaller part and two of the larger part; so that the proportion between the numbers of intervals<sup>49</sup> of the smaller part to the larger part and the length of the larger part to the length of the smaller part will be the same. Hence the product of the numbers of intervals of the minor part [the frequency] and the numbers [the frequency] of the major part will be equal to the product of the numbers of the major and the number of the minor part.<sup>50</sup> These products will therefore be: in the diapason [octave 2:1] the binary [2], in the diapente [3:2] the senary [6], in the diatessaron [4:3] the duodenary [12], in the major hexachord [5:3] the fortnightly [15], in the ditone [major third, 5:4] the vicenary [20], in the semiditone [minor third, 6:5] the tricenary [30], finally in the minor hexachord [minor sixth, 8:5] the quadricenary [40]: these numbers are not without an admirable regularity [mirabili analogia]\*.<sup>51</sup> Pleasure arises when the consonances reach the hearing because they relieve the senses, while on the contrary the pain derived from the dissonances is generated by their harshness, which can be easily experienced when the organ pipes are tuned. [283]

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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<sup>46</sup>That a shorter string moves faster than a longer is a common [possibly false] belief at the time.

<sup>47</sup>Force of traction.

<sup>48</sup>An interval of vibration is the duration of one cycle of the oscillations of the string, that is a period or a semi-period.

<sup>49</sup>The number of intervals, that is the number of periods or semi-periods, in a fixed time, is proportional to the frequency of vibration of the string.

<sup>50</sup>The number of a part is for its length. For example, in the chord divided into three parts there is a length ratio of 2:1 and a frequency ratio of 1:2. But the product of the lengths of the two parts and their frequencies is the same.

<sup>51</sup>In (Walker, 1978: p.31) and (Cohen, 1984: p. 269 note 7) a different translation of “analogia” is suggested, *proportion* instead of *regularity or logic*, which makes the interpretation of the various products 2, 6, 12, 15, etc., not as the order of agreeableness of consonances but simply a restatement of the ratios of the intervals. Of the long piece delimited by asterisk there is an alternative translation in Palisca (1994: pp. 215-216).

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