

Everything Is a Circle: A New Universal Orbital Model

Asli Pinar Tan

Independent Scientist in Electrical and Electronics Engineer, Istanbul, Türkiye

Email: aslipinartan@superonline.com

How to cite this paper: Tan, A.P. (2024) Everything Is a Circle: A New Universal Orbital Model. *Advances in Aerospace Science and Technology*, 9, 94-116. <https://doi.org/10.4236/aast.2024.93008>

Received: July 14, 2024

Accepted: September 16, 2024

Published: September 19, 2024

Copyright © 2024 by author(s) and Scientific Research Publishing Inc.

This work is licensed under the Creative Commons Attribution-NonCommercial International License (CC BY-NC 4.0).

<http://creativecommons.org/licenses/by-nc/4.0/>



Open Access

Abstract

Based on measured astronomical position data of heavenly objects in the Solar System and other planetary systems, all bodies in space seem to move in some kind of elliptical motion with respect to each other. According to Kepler's 1st Law, "orbit of a planet with respect to the Sun is an ellipse, with the Sun at one of the two foci." Orbit of the Moon with respect to Earth is also distinctly elliptical, but this ellipse has a varying eccentricity as the Moon comes closer to and goes farther away from the Earth in a harmonic style along a full cycle of this ellipse. In this paper, our research results are summarized, where it is first mathematically shown that the "distance between points around any two different circles in three-dimensional space" is equivalent to the "distance of points around a vector ellipse to another fixed or moving point, as in two-dimensional space". What is done is equivalent to showing that bodies moving on two different circular orbits in space vector-wise behave as if moving on an elliptical path with respect to each other, and virtually seeing each other as positioned at an instantaneously stationary point in space on their relative ecliptic plane, whether they are moving with the same angular velocity, or different but fixed angular velocities, or even with different and changing angular velocities with respect to their own centers of revolution. This mathematical revelation has the potential to lead to far reaching discoveries in physics, enabling more insight into forces of nature, with a formulation of a new fundamental model regarding the motions of bodies in the Universe, including the Sun, Planets, and Satellites in the Solar System and elsewhere, as well as at particle and subatomic level. Based on the demonstrated mathematical analysis, as they exhibit almost fixed elliptic orbits relative to one another over time, the assertion is made that the Sun, the Earth, and the Moon must each be revolving in their individual circular orbits of revolution in space. With this expectation, individual orbital parameters of the Sun, the Earth, and the Moon are calculated based on observed Earth to Sun and Earth to Moon distance data, also using analytical methods

developed as part of this research to an approximation. This calculation and analysis process have revealed additional results aligned with observation, and this also supports our assertion that the Sun, the Earth, and the Moon must actually be revolving in individual circular orbits.

Keywords

Solar System, Planetary System, Planet, Satellite, Earth, Moon, Topology, Circle, Sun, Ellipse, Orbit, Trajectory, Orbital Mechanics

1. Introduction

Based on measured astronomical position data of heavenly objects in the Solar System and other planetary systems, all bodies in space seem to move in some kind of elliptical motion with respect to each other. According to Kepler's 1st Law, "orbit of a planet with respect to the Sun is an ellipse, with the Sun at one of the two foci." Orbit of the Moon with respect to Earth is also distinctly elliptical, but this ellipse has a varying eccentricity as the Moon comes closer to and goes farther away from the Earth in a harmonic style along a full cycle of this ellipse. Existing theories of gravitation [1] have gaps in explaining all observed phenomena in completeness, which may be partly due to acceptance of this resultant elliptical mechanics that masks certain underlying physical behavior. An understanding of the true nature of orbital motion for moving objects in space may lead to a better formulation of gravitation theories, as well as all kinds of force interactions, including electromagnetics, weak and strong forces. We have formulated an alternative model of circular orbits that result in relative elliptic orbits, which may lead to a better understanding and formulation of forces in the Universe. Further, understanding the cycles of these individual circular orbits due to other planetary interactions may provide better insight into the major climate cycles over hundreds of thousands of years, the kind that lead to ice ages and extinction of species, as well as leading to a better understanding of the reason for observed phenomena such as the Earth's "axial precession" or the "precession of the equinoxes", "apsidal precession" of the Sun-Earth and Earth-Moon systems, change in the "eccentricity" of the observed Sun-Earth orbital ellipse, and the variation of Earth's obliquity with respect to the observed ecliptic.

In our published book [2] and data file [3], we have first mathematically derived in detail how the "distance between points around any two different circles in three-dimensional space" is equivalent to the "distance of points around a vector ellipse to another fixed or moving point, as in two-dimensional space", which is equivalent to showing that bodies moving on two different circular orbits in space vector-wise behave as if moving on an elliptical path with respect to each other, and virtually seeing each other as positioned at an instantaneously stationary point in space on their relative ecliptic plane, whether they are moving with the same angular velocity, or different but fixed angular velocities, or

even with different and changing angular velocities with respect to their own centers of revolution. Then, based on the demonstrated mathematical analysis, as they exhibit almost fixed elliptic orbits relative to one another over time, the assertion is made that the Sun, the Earth, and the Moon must each be revolving in their individual circular orbits of revolution in space. Further, based on this expectation, in our published book [2] and data file [3], individual orbital parameters of the Sun, the Earth, and the Moon are calculated based on observed Earth to Sun and Earth to Moon distance data, also using analytical methods developed as part of this research to an approximation. This calculation and analysis process have revealed additional results aligned with observation, and this also supports our assertion that the Sun, the Earth, and the Moon must actually be revolving in individual circular orbits.

In subsequent sections of this paper, our research results are summarized. This mathematical revelation has the potential to lead to far reaching discoveries in physics, enabling more insight into forces of nature, with a formulation of a new fundamental model regarding the motions of bodies in the Universe, including the Sun, Planets, and Satellites in the Solar System and elsewhere, as well as at particle and subatomic level.

2. Individual Circular Orbits Lead to Relative Elliptical Orbits

Consider a system of two circles in three dimensional space, with the geometry of the system in Cartesian $(\hat{x}, \hat{y}, \hat{z})$ coordinates depicted as in **Figure 1**, and the defining vectors expressed in the form of generic vector Equations (4)-(9). Points P_1 and P_2 are defined on these two circles, respectively, phased apart by a constant or time (t) -dependent angle ϕ_0 (3). Location of P_1 , phased at $[\phi_1(t) = \phi + \phi_0]$ (1) around its own circle, is defined by vector $[\vec{r}_1(\phi) + \vec{l}(\phi)]$ based on Equations (4)-(5) and Equation (8), and the location of P_2 , phased at $[\phi_2(t) = \phi]$ (2) around its own circle, is defined by vector $\vec{r}_2(\phi)$ (6).

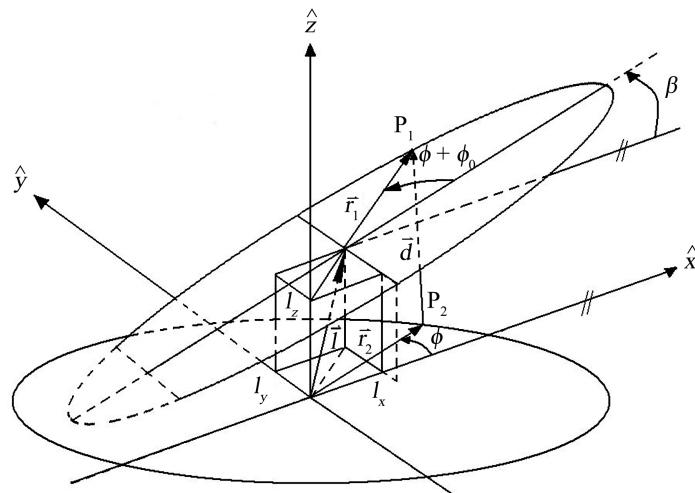


Figure 1. Distance between points P_1 and P_2 around two different circles in space.

$$\phi_1(t) = \phi + \phi_0 = \phi(t) + \phi_0(t) = \phi_2(t) + \phi_0(t) \quad (\text{Phase of } \mathbf{P}_1) \quad (1)$$

$$\phi_2(t) = \phi = \phi(t) \quad (\text{Phase of } \mathbf{P}_2) \quad (2)$$

$$\phi_0(t) = \phi_1(t) - \phi_2(t) = \phi_0 \quad (\text{Phase difference of } \mathbf{P}_1 \text{ and } \mathbf{P}_2) \quad (3)$$

At each phase ϕ (2), the two circles have vector radii $\bar{\mathbf{r}}_1$ (4)-(5) and $\bar{\mathbf{r}}_2$ (6) with constant magnitudes r_1 (7) and r_2 (7), respectively, and centers of these two circles are displaced by a constant or variable vector $\bar{\ell}(\phi)$ (8) with magnitude $\ell(\phi)$ (9), which is also the scalar distance between centers of the two circles.

Note that in all our subsequent operations, we take the “square of a vector” as the “dot product of the vector with itself”, which amounts to the scalar “square of the magnitude” for any vector.

$$\begin{aligned} \bar{\mathbf{r}}_1 &= \bar{\mathbf{r}}_1(\phi + \phi_0) \\ &= \hat{\mathbf{x}} r_1 \cos(\phi + \phi_0) \cos \beta + \hat{\mathbf{y}} r_1 \sin(\phi + \phi_0) + \hat{\mathbf{z}} r_1 \cos(\phi + \phi_0) \sin \beta \end{aligned} \quad (4)$$

$$\begin{aligned} \bar{\mathbf{r}}_1 &= \bar{\mathbf{r}}_1[\phi(t)] \\ &= (\hat{\mathbf{x}} r_1 \cos \beta \cos[\phi_0(t)] + \hat{\mathbf{y}} r_1 \sin[\phi_0(t)] \\ &\quad + \hat{\mathbf{z}} r_1 \sin \beta \cos[\phi_0(t)]) \cos[\phi(t)] + (-\hat{\mathbf{x}} r_1 \cos \beta \sin[\phi_0(t)] \\ &\quad + \hat{\mathbf{y}} r_1 \cos[\phi_0(t)] - \hat{\mathbf{z}} r_1 \sin \beta \sin[\phi_0(t)]) \sin[\phi(t)] \end{aligned} \quad (5)$$

$$\bar{\mathbf{r}}_2 = \bar{\mathbf{r}}_2[\phi(t)] = \hat{\mathbf{x}} r_2 \cos[\phi(t)] + \hat{\mathbf{y}} r_2 \sin[\phi(t)] \quad (6)$$

$$r_1 = |\bar{\mathbf{r}}_1| = \sqrt{\bar{\mathbf{r}}_1 \cdot \bar{\mathbf{r}}_1} = \sqrt{r_1^2}; \quad r_2 = |\bar{\mathbf{r}}_2| = \sqrt{\bar{\mathbf{r}}_2 \cdot \bar{\mathbf{r}}_2} = \sqrt{r_2^2} \quad (7)$$

$$\bar{\ell} = \bar{\ell}[\phi(t)] = \hat{\mathbf{x}} \ell_x[\phi(t)] + \hat{\mathbf{y}} \ell_y[\phi(t)] + \hat{\mathbf{z}} \ell_z[\phi(t)] \quad (8)$$

$$\begin{aligned} |\bar{\ell}(\phi)| &= \ell(\phi) = \sqrt{\bar{\ell}^2(\phi)} = \sqrt{\bar{\ell}(\phi) \cdot \bar{\ell}(\phi)} \\ \Rightarrow \ell^2(\phi) &= \bar{\ell}(\phi) \cdot \bar{\ell}(\phi) = \ell_x^2(\phi) + \ell_y^2(\phi) + \ell_z^2(\phi) \end{aligned} \quad (9)$$

The vector distance from any point \mathbf{P}_2 to \mathbf{P}_1 is expressed as $\bar{\mathbf{d}}(\phi)$ (10)-(12), where its magnitude $d(\phi)$ (13) is also the instantaneous scalar distance d (13) between points \mathbf{P}_1 and \mathbf{P}_2 on the two respective circles.

$$\begin{aligned} \bar{\mathbf{d}}(\phi) &= \bar{\mathbf{r}}_1(\phi + \phi_0) - \bar{\mathbf{r}}_2(\phi) + \bar{\ell}(\phi) = \bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2 + \bar{\ell} \\ &= \bar{\mathbf{a}} \cos \phi + \bar{\mathbf{b}} \sin \phi + \bar{\ell}(\phi) = \bar{\mathbf{X}}(\phi) + \bar{\mathbf{Y}}(\phi) + \bar{\ell}(\phi) \end{aligned} \quad (10)$$

$$\begin{aligned} \bar{\mathbf{d}}(\phi) &= \hat{\mathbf{x}} [r_1 \cos(\phi + \phi_0) \cos \beta - r_2 \cos \phi + \ell_x(\phi)] \\ &\quad + \hat{\mathbf{y}} [r_1 \sin(\phi + \phi_0) - r_2 \sin \phi + \ell_y(\phi)] \\ &\quad + \hat{\mathbf{z}} [r_1 \cos(\phi + \phi_0) \sin \beta + \ell_z(\phi)] \end{aligned} \quad (11)$$

$$\begin{aligned} \bar{\mathbf{d}}[\phi(t)] &= [\hat{\mathbf{x}} (r_1 \cos \beta \cos[\phi_0(t)] - r_2) + \hat{\mathbf{y}} r_1 \sin[\phi_0(t)] \\ &\quad + \hat{\mathbf{z}} r_1 \sin \beta \cos[\phi_0(t)]] \cos[\phi(t)] + [-\hat{\mathbf{x}} r_1 \cos \beta \sin[\phi_0(t)] \\ &\quad + \hat{\mathbf{y}} (r_1 \cos[\phi_0(t)] - r_2) - \hat{\mathbf{z}} r_1 \sin \beta \sin[\phi_0(t)]] \sin[\phi(t)] \\ &\quad + [\hat{\mathbf{x}} \ell_x[\phi(t)] + \hat{\mathbf{y}} \ell_y[\phi(t)] + \hat{\mathbf{z}} \ell_z[\phi(t)]] \end{aligned} \quad (12)$$

$$d(\phi) = |\bar{\mathbf{d}}(\phi)| = \sqrt{d^2(\phi)} = \sqrt{\bar{\mathbf{d}}(\phi) \cdot \bar{\mathbf{d}}(\phi)} = d \quad (13)$$

Vector distance $\bar{\mathbf{d}}(\phi)$ (12) at any phase ϕ (2) can equivalently be expressed as in $\bar{\mathbf{d}}(\phi)$ (10) in terms of virtual vectors $\bar{\mathbf{X}}(\phi)$ (14) and $\bar{\mathbf{Y}}(\phi)$ (15) that we have defined, with magnitudes are $X(\phi)$ (14) and $Y(\phi)$ (15), respectively, as well as vectors $\bar{\mathbf{a}}$ (16) and $\bar{\mathbf{b}}$ (17) we have defined, with magnitudes a (18) and b (19), respectively. When the phase difference ϕ_0 (3) is constant for all ϕ (2), a (18) and b (19) are also constant, but if $[\phi_0 = \phi_0(t)]$ (3) is time (t)-dependent, $a(t)$ (18) and $b(t)$ (19) are time (t)-dependent as well.

$$\begin{aligned} \bar{\mathbf{X}}(\phi) &= \bar{\mathbf{a}} \cos \phi \\ \bar{\mathbf{X}}(\phi) \cdot \bar{\mathbf{X}}(\phi) &= X^2(\phi) = \bar{\mathbf{a}} \cdot \bar{\mathbf{a}} \cos^2 \phi = a^2 \cos^2 \phi \\ |\bar{\mathbf{X}}(\phi)| &= X(\phi) \end{aligned} \quad (14)$$

$$\begin{aligned} \bar{\mathbf{Y}}(\phi) &= \bar{\mathbf{b}} \sin \phi \\ \bar{\mathbf{Y}}(\phi) \cdot \bar{\mathbf{Y}}(\phi) &= Y^2(\phi) = \bar{\mathbf{b}} \cdot \bar{\mathbf{b}} \sin^2 \phi = b^2 \sin^2 \phi \\ |\bar{\mathbf{Y}}(\phi)| &= Y(\phi) \end{aligned} \quad (15)$$

$$\begin{aligned} \bar{\mathbf{a}} &= \bar{\mathbf{a}}(t) \\ &= \hat{\mathbf{x}} \{r_1 \cos \beta \cos[\phi_0(t)] - r_2\} + \hat{\mathbf{y}} r_1 \sin[\phi_0(t)] + \hat{\mathbf{z}} r_1 \sin \beta \cos[\phi_0(t)] \\ &= (\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2)(\phi = 0) \end{aligned} \quad (16)$$

$$\begin{aligned} \bar{\mathbf{b}} &= \bar{\mathbf{b}}(t) \\ &= -\hat{\mathbf{x}} r_1 \cos \beta \sin[\phi_0(t)] + \hat{\mathbf{y}} \{r_1 \cos[\phi_0(t)] - r_2\} - \hat{\mathbf{z}} r_1 \sin \beta \sin[\phi_0(t)] \\ &= (\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2) \left(\phi = \frac{\pi}{2} \right) \end{aligned} \quad (17)$$

$$\begin{aligned} \bar{\mathbf{a}}(t) \cdot \bar{\mathbf{a}}(t) &= a^2(t) = r_1^2 - 2r_1 r_2 \cos \beta \cos[\phi_0(t)] + r_2^2 \\ a = a(t) &= |\bar{\mathbf{a}}(t)| = \sqrt{\bar{\mathbf{a}}(t) \cdot \bar{\mathbf{a}}(t)} \end{aligned} \quad (18)$$

$$\begin{aligned} \bar{\mathbf{b}}(t) \cdot \bar{\mathbf{b}}(t) &= b^2(t) = r_1^2 - 2r_1 r_2 \cos \beta \sin[\phi_0(t)] + r_2^2 \\ b = b(t) &= |\bar{\mathbf{b}}(t)| = \sqrt{\bar{\mathbf{b}}(t) \cdot \bar{\mathbf{b}}(t)} \end{aligned} \quad (19)$$

According to the definitions of vectors $\bar{\mathbf{X}}(\phi)$ (14), $\bar{\mathbf{Y}}(\phi)$ (15), $\bar{\mathbf{a}}$ (16), and $\bar{\mathbf{b}}$ (17), the relation in (20) is valid and holds for all ϕ (2) due to the given trigonometric identity in Equation (20). Therefore, the relation in Equation (20) reveals the validity of Equation (22) and Equation (21) for vector pair $[\bar{\mathbf{X}}(\phi), \bar{\mathbf{Y}}(\phi)]$ (14)-(15) and its magnitude pair $[X(\phi), Y(\phi)]$ (14)-(15), respectively.

$$\frac{\bar{\mathbf{X}}(\phi) \cdot \bar{\mathbf{X}}(\phi)}{\bar{\mathbf{a}} \cdot \bar{\mathbf{a}}} + \frac{\bar{\mathbf{Y}}(\phi) \cdot \bar{\mathbf{Y}}(\phi)}{\bar{\mathbf{b}} \cdot \bar{\mathbf{b}}} = \frac{X^2(\phi)}{a^2} + \frac{Y^2(\phi)}{b^2} = 1 = \cos^2 \phi + \sin^2 \phi \quad (20)$$

$$\boxed{\frac{X^2(\phi)}{a^2} + \frac{Y^2(\phi)}{b^2} = 1} \quad (21)$$

(Definition of Scalar Ellipse in 2-Dimensions)

$$\frac{\bar{\mathbf{X}}(\phi) \cdot \bar{\mathbf{X}}(\phi)}{\bar{\mathbf{a}} \cdot \bar{\mathbf{a}}} + \frac{\bar{\mathbf{Y}}(\phi) \cdot \bar{\mathbf{Y}}(\phi)}{\bar{\mathbf{b}} \cdot \bar{\mathbf{b}}} = 1 \quad (22)$$

(Definition of Vector Ellipse in 3-Dimensions)

As Equation (21) is the defining equation of an ellipse in 2-dimensions, where a (18) is the semi-major axis and b (19) is the semi-minor axis of the ellipse when $(a > b)$ (23), and vice versa, with Equation (21) reducing to the special case of a circle equation when $(a = b)$ (23), we claim Equation (22) indicates that vector pair $[\bar{\mathbf{X}}(\phi), \bar{\mathbf{Y}}(\phi)]$ (14)-(15) defines points on a vector ellipse in 3-dimensions in the most general case, $\bar{\mathbf{a}}$ (16) and $\bar{\mathbf{b}}$ (17) as its semi-major axis and semi-minor axis vectors. In other words, vector distance $\bar{\mathbf{d}}(\phi)$ (12) between two points \mathbf{P}_1 and \mathbf{P}_2 moving around two circles, whose centers are displaced by a constant or variable vector $\bar{\mathbf{l}}(\phi)$ (8), can equivalently be mathematically expressed and interpreted as the distance $\bar{\mathbf{d}}(\phi)$ (10) of points on a virtual vector ellipse, whose locations with respect to a virtual origin at each ϕ (2) are determined by the sum of the vector pair $[\bar{\mathbf{X}}(\phi), \bar{\mathbf{Y}}(\phi)]$ (14)-(15), from another point displaced from the same virtual origin of the ellipse by a constant or variable vector $[-\bar{\mathbf{l}}(\phi)]$ (8), where $\bar{\mathbf{a}}$ (16) and $\bar{\mathbf{b}}$ (17) are the fixed or variable semi-major axis semi-minor axis vectors of the vector ellipse. This result is mathematically valid even in the case when the phase difference $[\phi_0 = \phi_0(t)]$ (3) is a variable function of time (t). This revelation [2] [4] is the core and most significant finding of our research, whose detailed derivation is published in our book [2], with the data and calculations available in our file [3]. Moreover, we have also mathematically introduced the concept of a “vector ellipse” in Equation (22). More detail on our research results can be found in our book [2].

$$\begin{cases} a(t) > b(t) \Rightarrow \bar{\mathbf{a}}(t) \text{ is semi-major axis} \\ \quad \& \bar{\mathbf{b}}(t) \text{ is semi-minor axis of vector ellipse} \\ a(t) < b(t) \Rightarrow \bar{\mathbf{b}}(t) \text{ is semi-major axis} \\ \quad \& \bar{\mathbf{a}}(t) \text{ is semi-minor axis of vector ellipse} \\ a(t) = b(t) \Rightarrow \bar{\mathbf{a}}(t) \text{ and } \bar{\mathbf{b}}(t) \text{ are radii of vector circle} \end{cases} \quad (23)$$

Instantaneous focal distance c (24) of the vector ellipse can be determined using Equations (18)-(19), and instantaneous eccentricity e (25) of the vector ellipse can be found using Equations (18)-(19) and (24).

$$\begin{aligned} c &= c(t) = \sqrt{c^2(t)} = \sqrt{a^2(t) - b^2(t)} \\ &= \sqrt{2r_1r_2(1 - \cos \beta) |\cos[\phi_0(t)]|} \quad (\text{Focal Distance}) \end{aligned} \quad (24)$$

$$e = e(t) = \begin{cases} \frac{c(t)}{a(t)} = \sqrt{\frac{2r_1r_2(1 - \cos \beta) |\cos[\phi_0(t)]|}{r_1^2 - 2r_1r_2 \cos \beta \cos[\phi_0(t)] + r_2^2}} & \text{if } a(t) > b(t) \\ \frac{c(t)}{b(t)} = \sqrt{\frac{2r_1r_2(1 - \cos \beta) |\cos[\phi_0(t)]|}{r_1^2 - 2r_1r_2 \cos[\phi_0(t)] + r_2^2}} & \text{if } a(t) < b(t) \end{cases} \quad (\text{Eccentricity}) \quad (25)$$

The major consequence in physics, of this analysis and mathematical results in Equations (1) to (25) for moving points \mathbf{P}_1 and \mathbf{P}_2 on the two respective circles, is that “Particles or bodies moving around different circular orbits in space see themselves positioned on an elliptical route with respect to each other, these elliptical orbits having fixed or time-varying semi-major and semi-minor axes depending on the time dependencies of the angular velocities of the particles or bodies, where the particles or bodies instantaneously observe their counterpart virtually positioned at a fixed point in space, whose position is determined by the distance vector between centers of their individual circular orbits around which they revolve.”

This mathematical revelation has very far-reaching consequences in physics. It would lead to the formulation of a fundamental new model of motion for moving bodies in the Universe, including the Sun, the Planets, and the Satellites in the Solar System and elsewhere, as well as at particle and subatomic level, where “all bodies move at some angular velocity around their own circular orbits of revolution with different radii and centers of revolution in space”, but “each appear to be moving as if in elliptical orbits with respect to each other, where they see the other body located at a fixed or variable point in their virtual mutual plane of motion”. This subsequently would lead to more insight into forces in nature.

Based on observations and our findings as a result of our research, we make the broader assertion that “Moving bodies in space must be revolving around individual circular orbits within their revolution plane, which determine their individual equatorial planes”. The normal to its revolution plane and the direction of motion of the body around its individual circular orbit determine the axis of self-revolution according to “right hand rule”. The direction of self-rotation axis of each body in space is also determined according to “right hand rule” based on the direction of self-rotation of the body. Self-revolution axis and self-rotation axis of each moving body in space must be aligned along the normal of its equatorial plane. As all moving bodies in space must obey “Faraday’s Law of Induction”, we assert that all bodies in space possibly behave like macro scale charged particles moving in external magnetic fields with changing flux through the area of their individual circular orbit of revolution while moving in clockwise direction (Revolution Orbit “West” to Revolution Orbit “East”) from a vantage point above their North Pole, which we expect must be the main factor that triggers their self-rotation in the reverse (counterclockwise) direction (Self Rotation “West” to Self Rotation “East”) as seen from a vantage point above their North Pole, hence the bodies in the Universe have two Easts and two Wests in this context, where we always accept the North Pole of a body or a revolution to be the pole rotating in counterclockwise direction according to “right hand rule”. Based on this assertion, we also infer that the axial tilt of a body in its motion relative to another body must topologically be based on the orbital inclination angle between their planes of individual circular orbits of revolution in space. This assertion is also supported by our research results for the Sun-

Earth-Moon system presented in the following section, details of which can be found in our book [2].

3. Individual Circular Orbit Model Applied to Sun-Earth-Moon

Based on the demonstrated mathematical analysis, as they exhibit distinct elliptic orbits relative to one another over time, we make the assertion that Sun, Earth, and Moon must each be revolving in their individual circular orbits of revolution in space. With this expectation, orbital parameters of Sun, Earth, and Moon are calculated [2] [3] based on observed distance data between Sun and Earth, and Earth and Moon, and also using analytical methods developed as part of this research to an approximation, all of which are provided in detail in our published book [2] and data file [3], presented in Equations (26) to (57). Our analysis revealed additional results aligned with observation and thus supporting our assertion that Sun, Earth, and Moon must actually be revolving in individual circular orbits, and these can also be found in our book [2].

We have found [2] [3] out that the actual Sun-Earth-Moon topology in space can be represented roughly as in the configuration of **Figure 2**, not to scale, which in fact is the Sun-Earth configuration of **Figure 1** superimposed on the Earth-Moon configuration of **Figure 1**. Note that in the configuration of **Figure 2** with Cartesian $(\hat{x}, \hat{y}, \hat{z})$ coordinates, the plane of Earth's individual circular orbit of revolution is taken to be horizontal $\hat{x}-\hat{y}$ plane, and the direction of the individual circular orbit of revolution of Earth is taken to be counterclockwise in $[+\phi(t)]$ (2) direction looking down from \hat{z} -axis onto $\hat{x}-\hat{y}$ plane, and this corresponds to clockwise direction from a vantage point above the North Pole of the Earth, as our calculations have revealed that $(\hat{u}_{Earth\perp} = -\hat{z})$ (26) is the unit vector in the direction of self-rotation axis of Earth in this **Figure 2** configuration, which is also the direction of the North Pole of Earth.

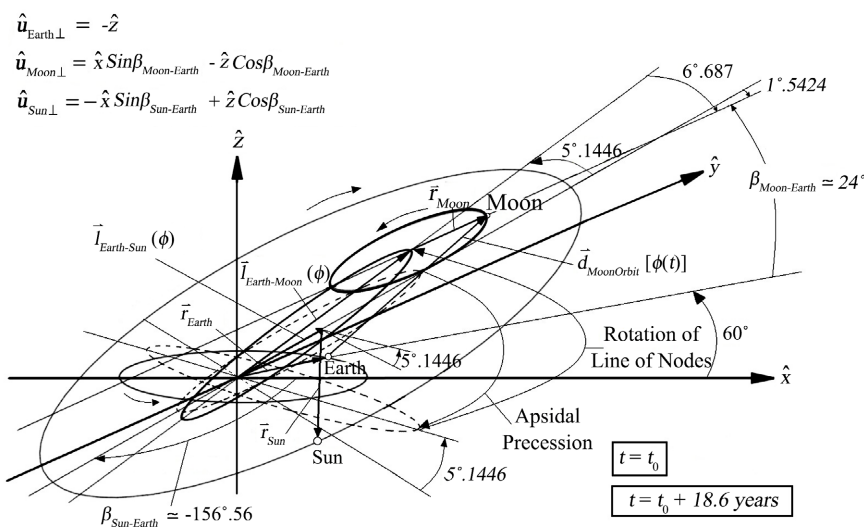


Figure 2. Sun-Earth-Moon configuration based on circular orbit model.

$$\hat{\mathbf{u}}_{Earth\perp} = -\hat{\mathbf{z}} \quad (\text{North Pole unit vector of Earth}) \quad (26)$$

The inclination angle of the plane of individual circular orbit of revolution of the Sun with respect to the plane of individual circular orbit revolution of the Earth is found as $\beta_{Sun-Earth}$ (27).

$$\begin{aligned} \beta_{Sun-Earth} &\approx -156.56^\circ \approx -2.732513127 \text{ radians} \\ &(\text{angle between Sun \& Earth revolution planes}) \end{aligned} \quad (27)$$

Our analysis based on observation has also led to the conclusion that the pole vector of the Sun that is angle-wise closer to the Earth's North Pole should be the Sun's South Pole vector. Subsequently, we have found the direction of the Sun's North Pole, also the unit vector in the direction of the self-rotation axis of the Sun, to be $(\hat{\mathbf{u}}_{Sun\perp} = -\hat{\mathbf{x}} \sin \beta_{Sun-Earth} + \hat{\mathbf{z}} \cos \beta_{Sun-Earth})$ (28) in the configuration of **Figure 2**. Furthermore, Sun's tilt with respect to the distance vector $[-\bar{\mathbf{d}}(\phi)]$ (10) from Sun to Earth is around 90° throughout a year according to our calculations, and we have also calculated the Sun's wobble angle and sideways swing angle as observed from Earth throughout a year in more detail in our book [2].

$$\begin{aligned} \hat{\mathbf{u}}_{Sun\perp} &= -\hat{\mathbf{x}} \sin \beta_{Sun-Earth} + \hat{\mathbf{z}} \cos \beta_{Sun-Earth} \\ &(\text{North Pole unit vector of Sun}) \end{aligned} \quad (28)$$

Our analysis has revealed that inclination angle of the plane of individual circular orbit of revolution of the Moon with respect to the plane of individual circular orbit revolution of Earth is $\beta_{Moon-Earth}$ (29), whereas the direction of self-rotation axis of the Moon, as is also the direction of North Pole of the Moon in the configuration of **Figure 2**, is $(\hat{\mathbf{u}}_{Moon\perp} = \hat{\mathbf{x}} \sin \beta_{Moon-Earth} - \hat{\mathbf{z}} \cos \beta_{Moon-Earth})$ (30).

$$\begin{aligned} \beta_{Moon-Earth} &\approx 24^\circ \\ &(\text{angle between Moon \& Earth revolution planes}) \end{aligned} \quad (29)$$

$$\begin{aligned} \hat{\mathbf{u}}_{Moon\perp} &= \hat{\mathbf{x}} \sin \beta_{Moon-Earth} - \hat{\mathbf{z}} \cos \beta_{Moon-Earth} \\ &(\text{North Pole unit vector of Moon}) \end{aligned} \quad (30)$$

We have found out that the constant phase difference $(\phi_{0,Earth-Sun} \approx -151.11^\circ)$ (31) between the Sun and the Earth in their individual orbits of revolution throughout a year, such that in its own cycle of individual revolution, Earth is moving ahead of the Sun in its own cycle of individual revolution by a phase difference of 151.11° (31). In Sun-Earth-Moon configuration of **Figure 2**, we have discovered that the phase $[\phi(t) = \phi_{Earth-Sun}]$ (32) of Earth in its individual circular orbit of revolution in its motion relative to the Sun in the Sun-Earth configuration of **Figure 1** at any date is about 60° ahead of the phase $[\phi(t) = \phi_{Earth-Moon}]$ (32) of Earth in its individual circular orbit of revolution in its motion relative to the Moon in the Earth-Moon configuration of **Figure 1**.

$$\begin{aligned} \phi_{0,Earth-Sun} &= -2.63736997198951 \text{ rad} = -151.11^\circ \\ &(\text{Phase difference for Sun-Earth system}) \end{aligned} \quad (31)$$

$$\phi_{Earth-Sun} \approx \phi_{Earth-Moon} + 60^\circ \quad (\text{at any date of the year}) \quad (32)$$

We have obtained the following radii magnitudes for r_{Sun} (33), r_{Earth} (34), and r_{Moon} (35), the individual circular orbits of revolution of the Sun, the Earth, and the Moon, respectively, in terms of kilometers (km) as well as astronomical units (au), based on observed yearly Earth to Sun distance data as well as Earth to Moon distance data, and using an analytical method we have developed to calculate orbital parameters of the heavenly bodies, to some approximations, all presented in detail in our book [1]. Even if we have obtained approximate values with the analytical calculation, the result we have reached is important in comparing the order of magnitude of the radii of individual circular orbits of revolution of Sun, Earth, and Moon, as $(r_{Earth} \ll r_{Moon} \ll r_{Sun})$ (36). It is also worth noting that r_{Earth} (34), the radius of individual orbital revolution of Earth, is about twice the radius of Earth itself.

$$r_{Sun} = \mathbf{1.00007869128446} \text{ au} = \mathbf{149609642.749} \text{ km} \quad (33)$$

(radius of Sun's revolution orbit)

$$r_{Earth} = \mathbf{0.0000826915340950721} \text{ au} = \mathbf{12370.477} \text{ km} \quad (34)$$

(radius of Earth's revolution orbit)

$$r_{Moon} \approx \mathbf{3302009.781} \text{ km} \approx \mathbf{0.0220725720604161} \text{ au} \quad (35)$$

(radius of Moon's revolution orbit)

$$(r_{Earth} \approx \mathbf{12370.477} \text{ km}) \ll (r_{Moon} \approx \mathbf{3302009.781} \text{ km}) \ll (r_{Sun} \approx \mathbf{149609642.749} \text{ km}) \quad (36)$$

The most significant and interesting implication based on the orbital parameter values found for the Sun, the Earth, and the Moon is, the obtained results indicate that the Earth is not revolving around the Sun, and the Moon is not directly revolving around the Earth, but the results in fact show that the Sun is revolving around a larger individual circular orbit which encompasses the individual circular orbits of the Earth and the Moon with an orbital inclination, where the centers of revolution of each are shifted with respect to each other's. The center of revolution of the Moon is in continuous harmonic motion relative to centers of revolution of other bodies, mainly over a cycle of approximately 18.6 years with respect to the center of Earth's revolution, which is analyzed in detail in our book [1]. Hence, we have concluded that planets do not necessarily revolve around stars, and Earth is not revolving around the Sun. These findings raise the need to reevaluate existing theories about formation, expected motional behavior, and topologies of solar systems observed in the Universe in general as well as our Solar System.

Phase values $[\phi(t) = \phi_{Earth-Sun}]$ (2) of the Earth for equinoxes, solstices, and dates of minimum and maximum Earth-Sun distances, as well as the dates of occurrence of some significant phases in the Earth-Sun yearly cycle are listed in Equations (37) to (46), in the Sun-Earth configuration of **Figure 1**.

$$\begin{aligned} \phi_{Earth-Sun, \text{Min Sun-Earth Distance}} &= \mathbf{-0.193236295254927} \text{ radians} \\ &= \mathbf{-11.072^\circ} \quad (\text{January 3}) \end{aligned} \quad (37)$$

$$\begin{aligned}\phi_{Earth-Sun} &= \mathbf{0.00058673254949726} \text{ radians} \\ &= \mathbf{0.034^\circ} \approx 0 \quad (\text{January 17})\end{aligned}\quad (38)$$

$$\begin{aligned}\phi_{Earth-Sun, Spring Equinox} &= \mathbf{1.02676771671574} \text{ radians} \\ &= \mathbf{58.829^\circ} \quad (\text{March 20})\end{aligned}\quad (39)$$

$$\begin{aligned}\phi_{Earth-Sun} &= \mathbf{1.57138305934439} \text{ radians} \\ &= \mathbf{90.034^\circ} \approx \frac{\pi}{2} \quad (\text{April 18})\end{aligned}\quad (40)$$

$$\begin{aligned}\phi_{Earth-Sun, Summer Solstice} &= -\phi_{0, Earth-Sun} = \mathbf{2.63736997198951} \text{ rad} \\ &= \mathbf{151.11^\circ} \quad (\text{June 21})\end{aligned}\quad (41)$$

$$\begin{aligned}\phi_{Earth-Sun, Max Sun-Earth Distance} &= \mathbf{2.94835635833487} \text{ radians} \\ &= \mathbf{168.928^\circ} \quad (\text{July 5})\end{aligned}\quad (42)$$

$$\begin{aligned}\phi_{Earth-Sun} &= \mathbf{3.14217938613929} \text{ radians} \\ &= \mathbf{180.034^\circ} \approx \pi \quad (\text{July 18})\end{aligned}\quad (43)$$

$$\begin{aligned}\phi_{Earth-Sun, Autumn Equinox} &= \mathbf{-2.05488176708229} \text{ radians} \\ &= \mathbf{242.264^\circ} = \mathbf{-117.736^\circ} \quad (\text{September 22})\end{aligned}\quad (44)$$

$$\begin{aligned}\phi_{Earth-Sun} &= \mathbf{-1.5702095942454} \text{ radians} \\ &= \mathbf{-89.966^\circ} \approx -\frac{\pi}{2} \quad (\text{October 17})\end{aligned}\quad (45)$$

$$\begin{aligned}\phi_{Earth-Sun, Winter Solstice} &= \pi - \phi_{0, Earth-Sun} = \mathbf{5.7789626255793} \text{ rad} \\ &= \mathbf{331.11^\circ} = \mathbf{-28.89^\circ} \quad (\text{December 21})\end{aligned}\quad (46)$$

For the relative motion ellipse of the Sun-Earth system, we have also determined the fixed semi-minor axis and semi-major axis vectors $[\bar{\mathbf{a}}(t) = \bar{\mathbf{a}}_{Earth-Sun}]$ (47) and $[\bar{\mathbf{b}}(t) = \bar{\mathbf{b}}_{Earth-Sun}]$ (48), respectively, as well as values of their fixed magnitudes $[a(t) = a_{Earth-Sun}]$ (49) and $[b(t) = b_{Earth-Sun}]$ (50), respectively, with their focal distance $[c(t) = c_{Earth-Sun}]$ (51) and eccentricity $[e(t) = e_{Earth-Sun}]$ (52), in the Sun-Earth configuration of **Figure 1**. It is notable that the eccentricity ($e_{Earth-Sun} = \mathbf{0.0166611047475858}$) (52) we have found for the relative Sun-Earth elliptical orbit based on our individual circular orbit model for the Sun and the Earth, using Sun-Earth distance data observed by Landsat, is the same as the 0.0167 eccentricity based on currently accepted elliptical model.

$$\begin{aligned}\bar{\mathbf{a}}_{Earth-Sun} &= \hat{\mathbf{x}} 120168467.040 \text{ km} - \hat{\mathbf{y}} 72280843.623 \text{ km} \\ &\quad + \hat{\mathbf{z}} 52106536.351 \text{ km}\end{aligned}\quad (47)$$

$$\begin{aligned}\bar{\mathbf{b}}_{Earth-Sun} &= -\hat{\mathbf{x}} 66316021.722 \text{ km} - \hat{\mathbf{y}} 131002922.948 \text{ km} \\ &\quad - \hat{\mathbf{z}} 28752488.898 \text{ km}\end{aligned}\quad (48)$$

$$\begin{aligned}a_{Earth-Sun} &= \mathbf{1.00001226586661} \text{ au} = \mathbf{149599705.647527} \text{ km} \\ &\quad (\text{Sun-Earth semi-minor axis magnitude})\end{aligned}\quad (49)$$

$$\begin{aligned}b_{Earth-Sun} &= \mathbf{1.00015109267839} \text{ au} = \mathbf{149620473.842965} \text{ km} \\ &\quad (\text{Sun-Earth semi-major axis magnitude})\end{aligned}\quad (50)$$

$$c_{Earth-Sun} = \mathbf{0.0166636221185271} \text{ au} = \mathbf{2492842.4} \text{ km} \quad (51)$$

(Focal distance of Sun-Earth system)

$$e_{Earth-Sun} = \frac{c_{Earth-Sun}}{b_{Earth-Sun}} = \mathbf{0.0166611047475858} \quad (52)$$

(Eccentricity of Sun-Earth system)

We have discovered that the magnitude $[\ell(\phi) = \ell_{Earth-Sun}(\phi)]$ (9) of the vector distance $\{\bar{\ell}[\phi(t)] = \bar{\ell}_{Earth-Sun}(\phi)\}$ (8) between the centers of individual circular orbits of revolutions of Earth and Sun exhibits three distinct main oscillation frequencies over a yearly Sun-Earth cycle. One oscillation is about 12 times a year, which is apparently based on the impact of the relative Earth-Moon motion. The second oscillation is seemingly based on the daily self-rotation of Earth around its own axis. The possible cause of the third oscillation, which is about 4 times over a Sun-Earth year, may be the impact of relative Sun-Mercury motion which has about four cycles over a Sun-Earth year. It is possible that the Sun and Mercury form a twin or binary system of bodies in space, in which Mercury behaves as the Sun's satellite, similar to Earth-Moon system, and this also implies the possibility that the individual circular orbits of revolution of the Sun and Mercury may be revolving around each other, to be confirmed when Mercury orbital parameters are determined. Our analysis also implies it is possible that relative motions of other planets and their satellites, and even other moving bodies in the Solar System, may impact the relative motion of Sun and Earth in terms of additional minor oscillations of $[\ell(\phi) = \ell_{Earth-Sun}(\phi)]$ (9), which can be observed only over longer periods than a Sun-Earth year.

Maximum distance $[\ell_{Earth-Sun}(\phi)]_{Max}$ (53) between centers of individual circular orbits of revolution of the Sun and the Earth occur on February 23, May 26, August 25, and November 24, and minimum distance $[\ell_{Earth-Sun}(\phi)]_{Min}$ (54) between centers of individual circular orbits of revolution of the Sun and the Earth occur on January 1, April 2, July 2, and October 1, over a yearly Sun-Earth cycle.

$$[\ell_{Earth-Sun}(\phi)]_{Max} = \mathbf{0.0168346184954446} \text{ au} \quad (53)$$

(23 February, 26 May, 25 August, 24 November)

$$[\ell_{Earth-Sun}(\phi)]_{Min} = \mathbf{0.0165381684638675} \text{ au} \quad (54)$$

(1 January, 2 April, 2 July, 1 October)

We have also determined values of time (t) varying semi major axis and semi minor axis vectors $[\bar{\mathbf{a}}(t) = \bar{\mathbf{a}}_{Earth-Moon}(t)]$ (55) and $[\bar{\mathbf{b}}(t) = \bar{\mathbf{b}}_{Earth-Moon}(t)]$ (56) of the relative vector ellipse of Earth-Moon system, and their instantaneous focal distance $[c(t) = c_{Earth-Moon}(t)]$ (57), to an approximation.

$$\begin{aligned} \bar{\mathbf{a}}_{Earth-Moon}(t) = & \hat{\mathbf{x}} [3016536.037 \cos(0.212768714989863t) - 12370.477] \text{ km} \\ & + \hat{\mathbf{y}} 3302009.781 \sin(0.212768714989863t) \text{ km} \\ & + \hat{\mathbf{z}} 1343048.374 \cos(0.212768714989863t) \text{ km} \end{aligned} \quad (55)$$

$$\begin{aligned} \bar{\mathbf{b}}_{Earth-Moon}(t) = & -\hat{\mathbf{x}} 3016536.037 \sin(0.212768714989863t) \text{ km} \\ & + \hat{\mathbf{y}} [3302009.781 \cos(0.212768714989863t) - 12370.477] \text{ km} \\ & - \hat{\mathbf{z}} 1343048.374 \sin(0.212768714989863t) \text{ km} \end{aligned} \quad (56)$$

$$c_{Earth-Moon}(t) = 7062892780.525 |\cos(0.212768714989863t)| \text{ km}^2 \quad (57)$$

(Focal Distance of Earth-Moon system)

Note here that in Equations (55)-(57), based on our analysis, the time (t) must be measured in [days] from a date that coincides with an Earth to Moon distance maximum in the month of March once in every 3 year cycles of Earth, such as March 18, 2017 and March 24, 2020.

Much more detail on our research results, such as about tilts, wobble angles, libration, and orientation variation cycles of the distance vector between Earth and Moon revolution centers, may be found in our book “Everything Is A Circle: A New Model For Orbits Of Bodies In The Universe” [2].

4. Interpretation of Kepler’s Laws, Classical Mechanics, Energy, and Angular Momentum in Terms of Individual Circular Orbit Model

We can further our analysis as follows to demonstrate how Kepler’s 2nd and 3rd laws of planetary motion can be interpreted in light of our individual circular orbit model, as well as the implications of our circular orbit model for classical mechanics, such as how our model accounts for angular momentum and energy.

In the configuration of our **Figure 1** and Equations (1)-(25), the angular velocity vectors $\bar{\omega}_1$ (58) (with period T_1 (58) and magnitude ω_1 (58)) and $\bar{\omega}_2$ (59) (with period T_2 (59) and magnitude ω_2 (59)) of the bodies revolving in uniform circular motion around vector radii $\bar{\mathbf{r}}_1$ (4)-(5) and $\bar{\mathbf{r}}_2$ (6) with constant magnitudes r_1 (7) and r_2 (7), respectively, have the associated tangential velocities $\bar{\mathbf{v}}_1$ (60) and $\bar{\mathbf{v}}_2$ (61), having constant magnitudes v_1 (60) and v_2 (61), respectively, where $\hat{\mathbf{v}}_1(\phi)$ (60) and $\hat{\mathbf{v}}_2(\phi)$ (61) are the unit vectors in the directions of $\bar{\mathbf{v}}_1$ (60) and $\bar{\mathbf{v}}_2$ (61), respectively. In the case when ϕ_0 (3) is constant, that is $(T_1 = T_2 = T)$ (62) as in the case of the Sun-Earth system, both bodies have the same angular velocity magnitude $(\omega = \omega_1 = \omega_2)$ (62).

$$\begin{aligned} \bar{\omega}_1 = \omega_1(-\hat{\mathbf{x}} \sin \beta + \hat{\mathbf{z}} \cos \beta) &= \frac{2\pi}{T_1}(-\hat{\mathbf{x}} \sin \beta + \hat{\mathbf{z}} \cos \beta) \\ \omega_1 = \frac{2\pi}{T_1} &= \frac{d(\phi + \phi_0)}{dt} \end{aligned} \quad (58)$$

$$\bar{\omega}_2 = \omega_2 \hat{\mathbf{z}} = \frac{2\pi}{T_2} \hat{\mathbf{z}}; \quad \omega_2 = \frac{2\pi}{T_2} = \frac{d\phi}{dt} \quad (59)$$

$$\begin{aligned} \bar{\mathbf{v}}_1 = \bar{\omega}_1 \times \bar{\mathbf{r}}_1 = \bar{\mathbf{v}}_1(\phi) &= \frac{d\bar{\mathbf{r}}_1}{dt} = \frac{d\bar{\mathbf{r}}_1}{d(\phi + \phi_0)} \frac{d(\phi + \phi_0)}{dt} = \frac{d\bar{\mathbf{r}}_1}{d(\phi + \phi_0)} \omega_1 \\ &= \frac{2\pi r_1}{T_1}(-\hat{\mathbf{x}} \sin \beta + \hat{\mathbf{z}} \cos \beta) \\ &\quad \times [\hat{\mathbf{x}} \cos(\phi + \phi_0) \cos \beta + \hat{\mathbf{y}} \sin(\phi + \phi_0) + \hat{\mathbf{z}} \cos(\phi + \phi_0) \sin \beta] \end{aligned}$$

$$\begin{aligned}
&= v_1 \left[-\hat{x} \sin(\phi + \phi_0) \cos \beta + \hat{y} \cos(\phi + \phi_0) - \hat{z} \sin(\phi + \phi_0) \sin \beta \right] \\
&= v_1 \hat{v}_1(\phi); \quad v_1 = \frac{2\pi r_1}{T_1} = r_1 \omega_1
\end{aligned} \tag{60}$$

$$\begin{aligned}
\bar{v}_2 &= \bar{\omega}_2 \times \bar{r}_2 = \bar{v}_2(\phi) = \frac{d\bar{r}_2}{dt} = \frac{d\bar{r}_2}{d\phi} \frac{d\phi}{dt} = \frac{d\bar{r}_2}{d\phi} \omega_2 \\
&= \left(\frac{2\pi}{T_2} \hat{z} \right) \times \left[\hat{x} r_2 \cos(\phi) + \hat{y} r_2 \sin(\phi) \right] \\
&= \frac{2\pi r_2}{T_2} \left[-\hat{x} \sin(\phi) + \hat{y} \cos(\phi) \right] \\
&= v_2 \left[-\hat{x} \sin(\phi) + \hat{y} \cos(\phi) \right] = v_2 \hat{v}_2(\phi); \quad v_2 = \frac{2\pi r_2}{T_2} = r_2 \omega_2
\end{aligned} \tag{61}$$

$$\begin{aligned}
\phi_0 \text{ constant} &\Rightarrow T_1 = T_2 = T \\
\Rightarrow \omega_1 &= \frac{2\pi}{T_1} = \frac{d(\phi + \phi_0)}{dt} = \frac{d\phi}{dt} = \frac{2\pi}{T_2} = \omega_2 = \frac{2\pi}{T} = \omega
\end{aligned} \tag{62}$$

In the configurations of our **Figure 1** and of **Figure 2**, the origin of the coordinate system is the center of individual circular revolution of the body at point \mathbf{P}_2 (Earth). Therefore, the positions $\bar{s}_1(\phi)$ and $\bar{s}_2(\phi)$, and velocities $\bar{v}_{\bar{s}_1}(\phi)$ and $\bar{v}_{\bar{s}_2}(\phi)$ of the bodies at \mathbf{P}_1 (Sun) and \mathbf{P}_2 (Earth) with respect to the origin can be expressed as in (63) and (64), respectively, where $\bar{v}_{\bar{\ell}}(\phi)$ (65) is the velocity of the $[\bar{\ell}(\phi) = \bar{\ell}_{\text{Earth-Sun}}(\phi)]$ (8) distance vector between centers of individual circular revolution orbits of the Earth and the Sun with magnitude $v_{\bar{\ell}}$ (65), ($\omega = \omega_1 = \omega_2$) (62) being the angular frequency of the Sun and the Earth in their revolutions around their individual circular orbits. Note that here $[\bar{\ell} \cdot \bar{v}_{\bar{\ell}}(\phi) = 0]$ (66) is expected to be true, as $\bar{v}_{\bar{\ell}}(\phi)$ (65) is a function of the derivative of $[\bar{\ell}(\phi) = \bar{\ell}_{\text{Earth-Sun}}(\phi)]$ (8) with respect to ϕ (2) and hence tangential to the $\bar{\ell}(\phi)$ (8) curve, meaning $\bar{v}_{\bar{\ell}}(\phi)$ (65) and $\bar{\ell}(\phi)$ (8) are perpendicular (\perp) to each other for all ϕ (2), similar to the tangential velocity and vector radius pairs of \bar{v}_1 (60) and \bar{r}_1 (4)-(5), and \bar{v}_2 (61) and \bar{r}_2 (6).

$$\bar{s}_1(\phi) = \bar{r}_1(\phi + \phi_0) + \bar{\ell}(\phi) = \bar{r}_1 + \bar{\ell}; \quad \bar{s}_2(\phi) = \bar{r}_2(\phi) = \bar{r}_2 \tag{63}$$

$$\bar{v}_{\bar{s}_1}(\phi) = \bar{v}_1(\phi) + \bar{v}_{\bar{\ell}}(\phi); \quad \bar{v}_{\bar{s}_2}(\phi) = \bar{v}_2(\phi) \tag{64}$$

$$\bar{v}_{\bar{\ell}}(\phi) = \frac{d\bar{\ell}}{dt} = \frac{d\bar{\ell}}{d\phi} \frac{d\phi}{dt} = \frac{d\bar{\ell}}{d\phi} \omega_1 = \frac{d\bar{\ell}}{d\phi} \frac{2\pi}{T_1} = \frac{d\bar{\ell}}{d\phi} \frac{2\pi}{T} = \frac{d\bar{\ell}}{d\phi} \omega \tag{65}$$

$$|\bar{v}_{\bar{\ell}}(\phi)| = v_{\bar{\ell}}(\phi)$$

$$\begin{aligned}
\left(\frac{d\bar{\ell}}{d\phi} \right) \perp \bar{\ell}(\phi) &\Rightarrow \bar{v}_{\bar{\ell}}(\phi) \perp \bar{\ell}(\phi) \\
&\Rightarrow \bar{\ell} \cdot \bar{v}_{\bar{\ell}}(\phi) = 0 \quad \text{for all } \phi
\end{aligned} \tag{66}$$

In alignment with Kepler's description of the Sun-Earth system, our two-body configuration revolving around their individual circular orbits can be reduced to that of one in which the observer on the body at \mathbf{P}_2 (Earth) revolving in uniform circular motion around its individual circular orbit with tangential velocity

\bar{v}_2 (61) could be relatively observed to be revolving with velocity $[\bar{v}_{21}(\phi) = \bar{v}_{s_2}(\phi) - \bar{v}_{s_1}(\phi)]$ (67) in an elliptical orbit at a varying vector distance $[-\bar{d}(\phi)]$ (10)-(12) around the stationary body at \mathbf{P}_1 (Sun). Symmetrically, this configuration could equivalently be interpreted in terms of the observer on the body at \mathbf{P}_1 (Sun) revolving in uniform circular motion around its individual circular orbit with tangential velocity \bar{v}_1 (60), relatively observed to be revolving with velocity $[\bar{v}_{12}(\phi) = \bar{v}_{s_1}(\phi) - \bar{v}_{s_2}(\phi)]$ (67) in an elliptical orbit at a varying vector distance $\bar{d}(\phi)$ (10)-(12) around the stationary body at \mathbf{P}_2 (Earth). However, as we are analyzing how Kepler's 2nd and 3rd laws of planetary motion can be interpreted in terms of our model, we will mainly concentrate on the relative Sun-centric case here.

$$\begin{aligned} [\bar{v}_{s_1}(\phi) - \bar{v}_{s_2}(\phi)] &= \bar{v}_{12}(\phi) = \bar{v}_1(\phi) + \bar{v}_\ell(\phi) - \bar{v}_2(\phi) \\ &= -\bar{v}_{21}(\phi) = -[\bar{v}_{s_2}(\phi) - \bar{v}_{s_1}(\phi)] \\ &= \hat{x}[-v_1 \sin(\phi + \phi_0) \cos \beta + v_2 \sin(\phi)] \\ &\quad + \hat{y}[v_1 \cos(\phi + \phi_0) - v_2 \cos(\phi)] \\ &\quad - \hat{z} v_1 \sin(\phi + \phi_0) \sin \beta + \bar{v}_\ell(\phi) \end{aligned} \tag{67}$$

Both $\bar{v}_{12}(\phi)$ (68) and $\bar{v}_{21}(\phi)$ (68) can be decomposed into their vector components that are parallel (\parallel) and perpendicular (\perp) to $\bar{d}(\phi)$ (10)-(12), in which case $\bar{v}_{12,\parallel\bar{d}}(\phi)$ (69) is the component of $\bar{v}_{12}(\phi)$ (68) parallel (\parallel) to $\bar{d}(\phi)$ (10)-(12), $\bar{v}_{12,\perp\bar{d}}(\phi)$ (70) is the component of $\bar{v}_{12}(\phi)$ (68) perpendicular (\perp) to $\bar{d}(\phi)$ (10)-(12), $\bar{v}_{21,\parallel(-\bar{d})}(\phi)$ (69) is the component of $\bar{v}_{21}(\phi)$ (68) parallel (\parallel) to $[-\bar{d}(\phi)]$ (10)-(12), and $\bar{v}_{21,\perp(-\bar{d})}(\phi)$ (70) is the component of $\bar{v}_{21}(\phi)$ (68) perpendicular (\perp) to $[-\bar{d}(\phi)]$ (10)-(12), utilizing the instantaneous scalar distance d (13) between points \mathbf{P}_1 (Sun) and \mathbf{P}_2 (Earth).

$$\begin{aligned} \bar{v}_{12}(\phi) &= \bar{v}_{12,\perp\bar{d}}(\phi) + \bar{v}_{12,\parallel\bar{d}}(\phi) \\ &= -[\bar{v}_{21,\perp(-\bar{d})}(\phi) + \bar{v}_{21,\parallel(-\bar{d})}(\phi)] \\ &= -\bar{v}_{21}(\phi) \end{aligned} \tag{68}$$

$$\begin{aligned} \bar{v}_{12,\parallel\bar{d}}(\phi) &= \frac{\bar{d}(\phi)}{d} \left\{ \frac{\bar{d}(\phi)}{d} \cdot [\bar{v}_{s_1}(\phi) - \bar{v}_{s_2}(\phi)] \right\} \\ &= -\frac{[-\bar{d}(\phi)]}{d} \left\{ \frac{[-\bar{d}(\phi)]}{d} \cdot [\bar{v}_{s_2}(\phi) - \bar{v}_{s_1}(\phi)] \right\} \\ &= -\bar{v}_{21,\parallel(-\bar{d})}(\phi) \end{aligned} \tag{69}$$

$$\begin{aligned} \bar{v}_{21,\perp(-\bar{d})}(\phi) &= [\bar{v}_{s_2}(\phi) - \bar{v}_{s_1}(\phi)] - \bar{v}_{21,\parallel(-\bar{d})}(\phi) \\ &= [\bar{v}_{s_2}(\phi) - \bar{v}_{s_1}(\phi)] + \frac{\bar{d}(\phi)}{d} \left\{ \frac{\bar{d}(\phi)}{d} \cdot [\bar{v}_{s_1}(\phi) - \bar{v}_{s_2}(\phi)] \right\} \end{aligned} \tag{70}$$

The dot product and cross product of the distance vector $\bar{d}(\phi)$ (10)-(12)

with $[\bar{\mathbf{v}}_{s_1}(\phi) - \bar{\mathbf{v}}_{s_2}(\phi)]$ (67) can be found in (71) and (73)-(74), respectively, for a two-body system of the same revolution period ($T_1 = T_2 = T$) (62) as in the Sun-Earth system case. Note that $[\bar{\ell} \cdot \bar{\mathbf{v}}_{\bar{\ell}}(\phi) = 0]$ (66) is utilized in (71). The magnitude $|\bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{s_1}(\phi) - \bar{\mathbf{v}}_{s_2}(\phi)]|$ (75) of the cross product equals

$[d(\phi)v_{12,\perp\bar{\mathbf{d}}}(\phi)]$ (75), as $[\bar{\mathbf{d}}(\phi) \times \bar{\mathbf{v}}_{12,\perp\bar{\mathbf{d}}}(\phi) = 0]$ due to the definition of cross product. Note that the relationship of symmetry in (72) also holds for the two bodies at \mathbf{P}_1 (Sun) and \mathbf{P}_2 (Earth).

For instance for an $[\bar{\ell}(\phi) = \bar{\ell}_{Earth-Sun}(\phi)]$ (8) vector of the simple example form in (76), the cross product of the distance vector $\bar{\mathbf{d}}(\phi)$ (10)-(12) with $[\bar{\mathbf{v}}_{s_1}(\phi) - \bar{\mathbf{v}}_{s_2}(\phi)]$ (67), namely $\{\bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{s_1}(\phi) - \bar{\mathbf{v}}_{s_2}(\phi)]\}$ (73) would take on a constant vector value as in (76) over a whole period of the relative two-body system of the same period ($T_1 = T_2 = T$) (62) as in Sun-Earth system case, and its magnitude $|\bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{s_1}(\phi) - \bar{\mathbf{v}}_{s_2}(\phi)]|$ (77) which is equal to

$[d(\phi)v_{21,\perp(-\bar{\mathbf{d}})}(\phi)]$ (75) takes on a constant value as in (77).

$$\begin{aligned} & \bar{\mathbf{d}}(\phi) \cdot [\bar{\mathbf{v}}_{s_1}(\phi) - \bar{\mathbf{v}}_{s_2}(\phi)] \\ &= [\bar{\mathbf{r}}_1(\phi + \phi_0) - \bar{\mathbf{r}}_2(\phi) + \bar{\ell}(\phi)] \cdot [\bar{\mathbf{v}}_1(\phi) + \bar{\mathbf{v}}_{\bar{\ell}}(\phi) - \bar{\mathbf{v}}_2(\phi)] \\ &= \{\hat{\mathbf{x}}[r_1 \cos(\phi + \phi_0) \cos \beta - r_2 \cos \phi + \ell_x(\phi)] + \hat{\mathbf{y}}[r_1 \sin(\phi + \phi_0) \\ &\quad - r_2 \sin \phi + \ell_y(\phi)] + \hat{\mathbf{z}}[r_1 \cos(\phi + \phi_0) \sin \beta + \ell_z(\phi)]\} \\ &\quad \cdot \frac{2\pi}{T} \{\hat{\mathbf{x}}[-r_1 \sin(\phi + \phi_0) \cos \beta + r_2 \sin \phi] + \hat{\mathbf{y}}[r_1 \cos(\phi + \phi_0) - r_2 \cos \phi] \\ &\quad - \hat{\mathbf{z}} r_1 \sin(\phi + \phi_0) \sin \beta + d\bar{\ell}/d\phi\} \\ &= \frac{2\pi}{T} \{r_1 r_2 (\cos \beta - 1) \sin(2\phi + \phi_0) + \ell_x(\phi) [-r_1 \sin(\phi + \phi_0) \cos \beta + r_2 \sin \phi] \\ &\quad + \ell_y(\phi) [r_1 \cos(\phi + \phi_0) - r_2 \cos \phi] - \ell_z(\phi) r_1 \sin(\phi + \phi_0) \sin \beta\} \\ &\quad + [\bar{\mathbf{r}}_1(\phi + \phi_0) - \bar{\mathbf{r}}_2(\phi)] \cdot \bar{\mathbf{v}}_{\bar{\ell}}(\phi) \end{aligned} \quad (71)$$

$$\bar{\mathbf{d}}(\phi) \cdot [\bar{\mathbf{v}}_{s_1}(\phi) - \bar{\mathbf{v}}_{s_2}(\phi)] = [-\bar{\mathbf{d}}(\phi)] \cdot [\bar{\mathbf{v}}_{s_2}(\phi) - \bar{\mathbf{v}}_{s_1}(\phi)] \quad (72)$$

$$\bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{s_1}(\phi) - \bar{\mathbf{v}}_{s_2}(\phi)] = [-\bar{\mathbf{d}}(\phi)] \times [\bar{\mathbf{v}}_{s_2}(\phi) - \bar{\mathbf{v}}_{s_1}(\phi)]$$

$$\begin{aligned} & \bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{s_1}(\phi) - \bar{\mathbf{v}}_{s_2}(\phi)] \\ &= [\bar{\mathbf{r}}_1(\phi + \phi_0) - \bar{\mathbf{r}}_2(\phi) + \bar{\ell}(\phi)] \times [\bar{\mathbf{v}}_1(\phi) - \bar{\mathbf{v}}_2(\phi) + \bar{\mathbf{v}}_{\bar{\ell}}(\phi)] \\ &= \{\hat{\mathbf{x}}[r_1 \cos(\phi + \phi_0) \cos \beta - r_2 \cos \phi + \ell_x(\phi)] \\ &\quad + \hat{\mathbf{y}}[r_1 \sin(\phi + \phi_0) - r_2 \sin \phi + \ell_y(\phi)] \\ &\quad + \hat{\mathbf{z}}[r_1 \cos(\phi + \phi_0) \sin \beta + \ell_z(\phi)]\} \\ &\quad \times \frac{2\pi}{T} \{\hat{\mathbf{x}}[-r_1 \sin(\phi + \phi_0) \cos \beta + r_2 \sin \phi] \\ &\quad + \hat{\mathbf{y}}[r_1 \cos(\phi + \phi_0) - r_2 \cos \phi] \\ &\quad - \hat{\mathbf{z}} r_1 \sin(\phi + \phi_0) \sin \beta + d\bar{\ell}/d\phi\} \end{aligned} \quad (73)$$

$$\begin{aligned}
 &\Rightarrow \bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{\bar{s}_1}(\phi) - \bar{\mathbf{v}}_{\bar{s}_2}(\phi)] \\
 &= \frac{2\pi}{T} \left\{ -\hat{\mathbf{x}} \left[r_1 \sin \beta (r_1 - r_2 \cos \phi_0) + \ell_y(\phi) r_1 \sin(\phi + \phi_0) \sin \beta \right. \right. \\
 &\quad \left. \left. + \ell_z(\phi) [r_1 \cos(\phi + \phi_0) - r_2 \cos \phi] \right] \right. \\
 &\quad \left. - [r_1 \sin(\phi + \phi_0) - r_2 \sin \phi + \ell_y(\phi)] (d\ell_z(\phi)/d\phi) \right. \\
 &\quad \left. + [r_1 \cos(\phi + \phi_0) \sin \beta + \ell_z(\phi)] (d\ell_y(\phi)/d\phi) \right\} \\
 &\quad + \hat{\mathbf{y}} \left\{ -r_1 r_2 \sin \beta \sin \phi_0 + \ell_x(\phi) r_1 \sin(\phi + \phi_0) \sin \beta \right. \\
 &\quad \left. + \ell_z(\phi) [-r_1 \sin(\phi + \phi_0) \cos \beta + r_2 \sin \phi] \right. \\
 &\quad \left. + [r_1 \cos(\phi + \phi_0) \sin \beta + \ell_z(\phi)] (d\ell_x(\phi)/d\phi) \right. \\
 &\quad \left. - [r_1 \cos(\phi + \phi_0) \cos \beta - r_2 \cos \phi + \ell_x(\phi)] (d\ell_z(\phi)/d\phi) \right\} \\
 &\quad + \hat{\mathbf{z}} \left\{ r_1^2 \cos \beta - r_1 r_2 (\cos \beta + 1) \cos \phi_0 + r_2^2 \right. \\
 &\quad \left. + \ell_x(\phi) [r_1 \cos(\phi + \phi_0) - r_2 \cos \phi] \right. \\
 &\quad \left. - \ell_y(\phi) [-r_1 \sin(\phi + \phi_0) \cos \beta + r_2 \sin \phi] \right. \\
 &\quad \left. + [r_1 \cos(\phi + \phi_0) \cos \beta - r_2 \cos \phi + \ell_x(\phi)] (d\ell_y(\phi)/d\phi) \right. \\
 &\quad \left. - [r_1 \sin(\phi + \phi_0) - r_2 \sin \phi + \ell_y(\phi)] (d\ell_x(\phi)/d\phi) \right\} \tag{74}
 \end{aligned}$$

$$\begin{aligned}
 &|\bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{\bar{s}_1}(\phi) - \bar{\mathbf{v}}_{\bar{s}_2}(\phi)]| \\
 &= |\bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{12,\perp\bar{a}}(\phi) + \bar{\mathbf{v}}_{12,\parallel\bar{a}}(\phi)]| \\
 &= \left| [\bar{\mathbf{d}}(\phi) \times \bar{\mathbf{v}}_{12,\perp\bar{a}}(\phi)] + [\bar{\mathbf{d}}(\phi) \times \bar{\mathbf{v}}_{12,\parallel\bar{a}}(\phi)] \right| \tag{75} \\
 &= |\bar{\mathbf{d}}(\phi) \times \bar{\mathbf{v}}_{12,\perp\bar{a}}(\phi)| \\
 &= d(\phi) v_{12,\perp\bar{a}}(\phi) = d(\phi) v_{21,\perp(-\bar{a})}(\phi)
 \end{aligned}$$

$$\bar{\ell}(\phi) = \hat{\mathbf{x}} \ell_x(\phi) + \hat{\mathbf{y}} \ell_y(\phi) + \hat{\mathbf{z}} \ell_z(\phi); \quad \ell_1 \text{ constant}$$

$$\text{If } \begin{cases} \ell_x(\phi) = -\ell_1 \cos \phi \cos \beta \\ \ell_y(\phi) = \ell_1 \sin \phi \\ \ell_z(\phi) = -\ell_1 \cos \phi \sin \beta \end{cases}$$

$$\Rightarrow \bar{\mathbf{v}}_{\bar{\ell}}(\phi) = \left(\frac{2\pi}{T} \right) \frac{d\bar{\ell}}{d\phi} = \frac{2\pi}{T} (\hat{\mathbf{x}} \ell_1 \sin \phi \cos \beta + \hat{\mathbf{y}} \ell_1 \cos \phi + \hat{\mathbf{z}} \ell_1 \sin \phi \sin \beta)$$

$$\begin{aligned}
 &\Rightarrow \bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{\bar{s}_1}(\phi) - \bar{\mathbf{v}}_{\bar{s}_2}(\phi)] = \text{constant for all } \phi \\
 &= \frac{2\pi}{T} \left\{ -\hat{\mathbf{x}} \sin \beta [r_1 (r_1 - r_2 \cos \phi_0) + (r_2 \ell_1 - \ell_1^2)] - \hat{\mathbf{y}} r_1 r_2 \sin \beta \sin \phi_0 \right. \\
 &\quad \left. + \hat{\mathbf{z}} [r_1^2 \cos \beta - r_1 r_2 (\cos \beta + 1) \cos \phi_0 + r_2^2] \right. \\
 &\quad \left. - (\ell_1^2 \cos \beta - r_2 \ell_1 \cos \beta + r_2 \ell_1) \right\} \tag{76} \\
 &\Rightarrow \bar{\mathbf{d}}(\phi) \cdot [\bar{\mathbf{v}}_{\bar{s}_1}(\phi) - \bar{\mathbf{v}}_{\bar{s}_2}(\phi)] \\
 &= \frac{2\hat{\mathbf{z}}}{T} \{ r_1 r_2 (\cos \beta - 1) \sin(2\phi + \phi_0) \\
 &\quad + r_1 \ell_1 [2 \sin(2\phi + \phi_0) - \sin(2\phi) (\cos \beta + 1)] \}
 \end{aligned}$$

$$\begin{aligned}
& \text{If } \begin{cases} \ell_x(\phi) = -\ell_1 \cos \phi \cos \beta \\ \ell_y(\phi) = \ell_1 \sin \phi \\ \ell_z(\phi) = -\ell_1 \cos \phi \sin \beta \end{cases} \\
& \Rightarrow \left| \vec{d}(\phi) \times [\vec{v}_{s_1}(\phi) - \vec{v}_{s_2}(\phi)] \right| \\
& = d(\phi) v_{12,\perp\vec{d}}(\phi) = d(\phi) v_{21,\perp(-\vec{d})}(\phi) = \text{constant for all } \phi \quad (77) \\
& = \frac{2\pi}{T} \sqrt{\sin^2 \beta \left\{ \left[r_1 (r_1 - r_2 \cos \phi_0) + (r_2 \ell_1 - \ell_1^2) \right]^2 + r_1^2 r_2^2 \sin^2 \phi_0 \right\} + \left[(r_1^2 \cos \beta - r_1 r_2 (\cos \beta + 1) \cos \phi_0 + r_2^2) \right. \\
& \quad \left. - (\ell_1^2 \cos \beta - r_2 \ell_1 \cos \beta + r_2 \ell_1) \right]^2}
\end{aligned}$$

Note that according to this two-body system where both bodies are revolving (not stationary) according to our circular orbit model, at every incremental time interval (dt) (62), the body at \mathbf{P}_1 (Sun) traverses an angular displacement of ($d\phi$) (62) in its individual orbit, and the body at \mathbf{P}_2 (Earth) traverses an angular displacement of ($d\phi$) (62) in its individual orbit, but the vector $\vec{d}(\phi)$ (10)-(12) of magnitude (instantaneous scalar distance) d (13) between points \mathbf{P}_1 (Sun) and \mathbf{P}_2 (Earth) traverses a different angular displacement ($d\theta$) (78) based on the magnitude $v_{21,\perp(-\vec{d})}(\phi)$ (78) of $\vec{v}_{12,\perp\vec{d}}(\phi) = -\vec{v}_{21,\perp(-\vec{d})}(\phi)$ (68) component of $\{\vec{v}_{s_1}(\phi) - \vec{v}_{s_2}(\phi)\}$ (67), which is perpendicular (\perp) to distance vector $\vec{d}(\phi)$ (10)-(12) between the two bodies. Therefore, when the Sun and the Earth complete one yearly cycle of period ($T_1 = T_2 = T$) (62) in their individual orbits, although they each traverse angular displacements of 2π radians in their individual circular orbits, the distance vector $\vec{d}(\phi)$ (10)-(12) between them would traverse an angular displacement *different* from 2π radians in 3-dimensional space over a year. When an incremental time interval (dt) (62) passes, $\vec{d}(\phi)$ (10)-(12) would traverse an incremental area of dA (79) in the relative Sun-Earth elliptical orbit, and the area swept by $\vec{d}(\phi)$ (10)-(12) per unit time can be expressed as in (79), which must be constant according to Kepler's 2nd law. This would hold for the Sun-Earth system case if the

$[\vec{\ell}(\phi) = \vec{\ell}_{Earth-Sun}(\phi)]$ (8) distance vector between the centers of individual circular revolution orbits of Earth and Sun take on a form such as the example in (76) even if more complicated, in which case the cross product of distance vector $\vec{d}(\phi)$ (10)-(12) with $[\vec{v}_{s_1}(\phi) - \vec{v}_{s_2}(\phi)]$ (67), namely

$\{\vec{d}(\phi) \times [\vec{v}_{s_1}(\phi) - \vec{v}_{s_2}(\phi)]\}$ (73) would take on a constant vector value similar to the case in (76) over a whole period of the relative two-body system of the same period ($T_1 = T_2 = T$) (62), and its magnitude $|\vec{d}(\phi) \times [\vec{v}_{s_1}(\phi) - \vec{v}_{s_2}(\phi)]|$ (77) which is equal to $[d(\phi) v_{21,\perp(-\vec{d})}(\phi)]$ (75) would take on a constant value as in (77), in alignment with Kepler's 2nd law of equal areas that can be rephrased as "the line $\vec{d}(\phi)$ (10)-(12) between the Sun and the planet sweeps equal areas dA (79) in equal times (dt) (62), thus speed of the planet $v_{21,\perp(-\vec{d})}(\phi)$ (75)

increases as it nears the Sun and decreases as it recedes from the Sun.”

$$d(d\theta) = v_{21,\perp(-\bar{a})}(\phi)(dt) \Rightarrow d\left(\frac{d\theta}{dt}\right) = v_{21,\perp(-\bar{a})}(\phi) \quad (78)$$

$$(dA) = d\frac{d}{2}(d\theta) = \frac{d^2}{2}(d\theta) \Rightarrow \left(\frac{dA}{dt}\right) = \frac{d^2}{2}\left(\frac{d\theta}{dt}\right) = \frac{d}{2}v_{21,\perp(-\bar{a})}(\phi) \quad (79)$$

The body at \mathbf{P}_1 (Sun) with mass m_1 would have a constant angular momentum vector $\bar{\mathbf{L}}_1(\phi)$ (80) of constant magnitude L_1 (80) in its individual circular orbit, and the body at \mathbf{P}_2 (Earth) with mass m_2 would have a constant angular momentum vector $\bar{\mathbf{L}}_2(\phi)$ (81) of constant magnitude L_2 (81) in its individual circular orbit.

$$\bar{\mathbf{L}}_1(\phi) = m_1[\bar{\mathbf{r}}_1(\phi) \times \bar{\mathbf{v}}_1(\phi)] = m_1 r_1^2 \frac{2\pi}{T} (-\hat{\mathbf{x}} \sin \beta + \hat{\mathbf{z}} \cos \beta) \quad (80)$$

$$L_1(\phi) = m_1 r_1^2 \frac{2\pi}{T} = L_1$$

$$\bar{\mathbf{L}}_2(\phi) = m_2[\bar{\mathbf{r}}_2(\phi) \times \bar{\mathbf{v}}_2(\phi)] = m_2 r_2^2 \frac{2\pi}{T} \hat{\mathbf{z}} \quad (81)$$

$$L_2(\phi) = m_2 r_2^2 \frac{2\pi}{T} = L_2$$

On the other hand, according to our circular orbital model, angular momentum of the body at \mathbf{P}_1 (Sun) with respect to its motion relative to the body at \mathbf{P}_2 (Earth) would be $\bar{\mathbf{L}}_{12,\perp\bar{a}}(\phi)$ (82) with magnitude $L_{12,\perp\bar{a}}(\phi) = m_1 d v_{12,\perp\bar{a}}(\phi)$ (83), and the angular momentum of the body at \mathbf{P}_2 (Earth) with respect to its motion relative to the body at \mathbf{P}_1 (Sun) would be $\bar{\mathbf{L}}_{21,\perp(-\bar{a})}(\phi)$ (82) with magnitude $L_{21,\perp(-\bar{a})}(\phi) = m_2 d v_{21,\perp(-\bar{a})}(\phi)$ (83). We also find that the specific angular momentum for the two bodies relative to each other is the same (84).

$$\begin{aligned} \bar{\mathbf{L}}_{12,\perp\bar{a}}(\phi) &= m_1 \bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{\bar{s}_1}(\phi) - \bar{\mathbf{v}}_{\bar{s}_2}(\phi)] \\ \bar{\mathbf{L}}_{21,\perp(-\bar{a})}(\phi) &= m_2 [-\bar{\mathbf{d}}(\phi)] \times [\bar{\mathbf{v}}_{\bar{s}_2}(\phi) - \bar{\mathbf{v}}_{\bar{s}_1}(\phi)] \end{aligned} \quad (82)$$

$$\begin{aligned} \frac{|\bar{\mathbf{L}}_{12,\perp\bar{a}}(\phi)|}{m_1} &= \frac{L_{12,\perp\bar{a}}(\phi)}{m_1} = |\bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{\bar{s}_1}(\phi) - \bar{\mathbf{v}}_{\bar{s}_2}(\phi)]| \\ &= d(\phi) v_{12,\perp\bar{a}}(\phi) = 2 \left(\frac{dA}{dt}\right) = d(\phi) v_{21,\perp(-\bar{a})}(\phi) \\ &= |[-\bar{\mathbf{d}}(\phi)] \times [\bar{\mathbf{v}}_{\bar{s}_2}(\phi) - \bar{\mathbf{v}}_{\bar{s}_1}(\phi)]| \\ &= \frac{L_{21,\perp(-\bar{a})}(\phi)}{m_2} = \frac{|\bar{\mathbf{L}}_{21,\perp(-\bar{a})}(\phi)|}{m_2} \end{aligned} \quad (83)$$

$$\begin{aligned} \frac{\bar{\mathbf{L}}_{12,\perp\bar{a}}(\phi)}{m_1} &= \bar{\mathbf{d}}(\phi) \times [\bar{\mathbf{v}}_{\bar{s}_1}(\phi) - \bar{\mathbf{v}}_{\bar{s}_2}(\phi)] \\ &= [-\bar{\mathbf{d}}(\phi)] \times [\bar{\mathbf{v}}_{\bar{s}_2}(\phi) - \bar{\mathbf{v}}_{\bar{s}_1}(\phi)] = \frac{\bar{\mathbf{L}}_{21,\perp(-\bar{a})}(\phi)}{m_2} \end{aligned} \quad (84)$$

According to our circular model and classical mechanics, the total energy of this two-body system pertaining to only the revolutions and interaction of the

two bodies would be E_{total} (85), not including their self-rotations.

$$\begin{aligned}
E_{total} &= K_1 + K_2 + U(\vec{d}) \\
&= \frac{1}{2}m_1 |\vec{v}_{\bar{s}_1}(\phi)|^2 + \frac{1}{2}m_2 |\vec{v}_{\bar{s}_2}(\phi)|^2 + U(\vec{d}(\phi)) \\
&= \frac{1}{2}m_1 [\vec{v}_{\bar{s}_1}(\phi) \cdot \vec{v}_{\bar{s}_1}(\phi)] + \frac{1}{2}m_2 [\vec{v}_{\bar{s}_2}(\phi) \cdot \vec{v}_{\bar{s}_2}(\phi)] - \frac{Gm_1m_2}{d(\phi)} \\
&= \frac{1}{2}m_1 [v_1^2 + 2\vec{v}_1(\phi) \cdot \vec{v}_{\bar{t}}(\phi) + v_{\bar{t}}^2] + \frac{1}{2}m_2 v_2^2 - \frac{Gm_1m_2}{d(\phi)} \quad (85) \\
&= \frac{1}{2}(m_1 + m_2) \left[\frac{m_1 \vec{v}_{\bar{s}_1}(\phi) + m_2 \vec{v}_{\bar{s}_2}(\phi)}{m_1 + m_2} \right]^2 \\
&\quad + \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} [\vec{v}_{\bar{s}_1}(\phi) - \vec{v}_{\bar{s}_2}(\phi)]^2 - \frac{Gm_1m_2}{d(\phi)} \\
&= K_{barycenter} + K_{\mu} - \frac{Gm_1m_2}{d(\phi)}
\end{aligned}$$

K_1 (86) is the kinetic energy of the body at \mathbf{P}_1 (Sun), whereas K_2 (87) is the kinetic energy of the body at \mathbf{P}_2 (Earth). $U(\vec{d}(\phi))$ (88) is the potential energy of this two-body system separated at any instant from each other by the distance vector $\vec{d}(\phi)$ (10)-(12), which is replaced here by the gravitational potential energy in analogy of Kepler's planetary system configuration, G taken to be the gravitational constant. However, the potential energy $U(\vec{d}(\phi))$ (88) could be of any kind (e.g. electromagnetic as well) based on the setting and scale.

$$\begin{aligned}
K_1 &= \frac{1}{2}m_1 |\vec{v}_{\bar{s}_1}(\phi)|^2 = \frac{1}{2}m_1 [\vec{v}_{\bar{s}_1}(\phi) \cdot \vec{v}_{\bar{s}_1}(\phi)] \\
&= \frac{1}{2}m_1 [v_1^2 + 2\vec{v}_1(\phi) \cdot \vec{v}_{\bar{t}}(\phi) + v_{\bar{t}}^2] \quad (86)
\end{aligned}$$

$$K_2 = \frac{1}{2}m_2 |\vec{v}_{\bar{s}_2}(\phi)|^2 = \frac{1}{2}m_2 [\vec{v}_{\bar{s}_2}(\phi) \cdot \vec{v}_{\bar{s}_2}(\phi)] = \frac{1}{2}m_2 v_2^2 \quad (87)$$

$$U(\vec{d}) = U(\vec{d}(\phi)) = -\frac{Gm_1m_2}{d(\phi)} \quad (88)$$

$K_{barycenter}$ (89) is the kinetic energy of the barycenter (center of mass) of this two-body (e.g. Sun-Earth) system, and K_{μ} (90) is the kinetic energy of reduced mass μ (91) for this two-body (e.g. Sun-Earth) system.

$$K_{barycenter} = \frac{1}{2}(m_1 + m_2) \left[\frac{m_1 \vec{v}_{\bar{s}_1}(\phi) + m_2 \vec{v}_{\bar{s}_2}(\phi)}{m_1 + m_2} \right]^2 \quad (89)$$

$$K_{\mu} = \frac{1}{2}\mu [\vec{v}_{\bar{s}_1}(\phi) - \vec{v}_{\bar{s}_2}(\phi)]^2 = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} [\vec{v}_{\bar{s}_1}(\phi) - \vec{v}_{\bar{s}_2}(\phi)]^2 \quad (90)$$

$$\mu = \frac{m_1 m_2}{(m_1 + m_2)} = \frac{1}{\frac{1}{m_1} + \frac{1}{m_2}} \quad (\text{reduced mass}) \quad (91)$$

For our Sun-Earth system which is a relative Keplerian orbit, the vis-viva equation [5] can be expressed [6] as in (92), which leads to the expression of

specific orbital energy [7] \in (93), where $[\bar{v}_{21}(\phi) = \bar{v}_{s_2}(\phi) - \bar{v}_{s_1}(\phi)]$ (67) is the relative velocity of the body at \mathbf{P}_2 (Earth) with respect the body at \mathbf{P}_1 (Sun). As $\bar{\mathbf{b}}_{Earth-Sun}$ (48) is the semi-major axis vector of the relative Sun-Earth orbital ellipse which occurs at $(\phi = \pi/2)$, $\bar{\mathbf{b}} = (\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2)(\phi = \pi/2)$ (17) is the semi-major axis vector with magnitude $b = \sqrt{r_1^2 - 2r_1r_2 \cos \phi_0 + r_2^2}$ (19), ϕ_0 (3) and b (19) being constant for the Sun-Earth system.

$$[\bar{v}_{s_2}(\phi) - \bar{v}_{s_1}(\phi)]^2 = G(m_1 + m_2) \left[\frac{2}{d(\phi)} - \frac{1}{b} \right] \quad (92)$$

$$\in = \frac{K_\mu + U(\bar{\mathbf{d}})}{\mu} = \frac{1}{2} [\bar{v}_{s_2}(\phi) - \bar{v}_{s_1}(\phi)]^2 - \frac{G(m_1 + m_2)}{d(\phi)} = -\frac{G(m_1 + m_2)}{2b} \quad (93)$$

Expanding the equation in (93) as in (94), and as specific total energy \in (93) is constant throughout the orbit, equating the two sides at $(\phi = \pi/2)$ yields (95), utilizing the definition in $\bar{\mathbf{d}}(\phi)$ (10).

$$\begin{aligned} & [\bar{v}_{s_2}(\phi) - \bar{v}_{s_1}(\phi)]^2 = [\bar{v}_1(\phi) + \bar{v}_\ell(\phi) - \bar{v}_2(\phi)]^2 = \frac{2Gm_1}{d(\phi)} - \frac{Gm_1}{b} \\ & = \bar{v}_1(\phi) \cdot \bar{v}_1(\phi) + 2\bar{v}_1(\phi) \cdot \bar{v}_\ell(\phi) + \bar{v}_\ell(\phi) \cdot \bar{v}_\ell(\phi) \\ & \quad - 2\bar{v}_1(\phi) \cdot \bar{v}_2(\phi) - 2\bar{v}_\ell(\phi) \cdot \bar{v}_2(\phi) + \bar{v}_2(\phi) \cdot \bar{v}_2(\phi) \\ & = v_1^2 + v_\ell^2(\phi) + v_2^2 + 2[\bar{v}_1(\phi) \cdot \bar{v}_\ell(\phi) - \bar{v}_1(\phi) \cdot \bar{v}_2(\phi) - \bar{v}_\ell(\phi) \cdot \bar{v}_2(\phi)] \quad (94) \\ & = 4\pi^2 \left\{ \frac{r_1^2}{T_1^2} + \frac{1}{T_1^2} \left(\frac{d\bar{\ell}}{d\phi} \right) \cdot \left(\frac{d\bar{\ell}}{d\phi} \right) + \frac{r_2^2}{T_2^2} + 2 \left[\frac{r_1}{T_1} \hat{\mathbf{v}}_1(\phi) \cdot \left(\frac{d\bar{\ell}}{d\phi} \right) \right. \right. \\ & \quad \left. \left. - \frac{r_2}{T_1 T_2} \left(\frac{d\bar{\ell}}{d\phi} \right) \cdot \hat{\mathbf{v}}_2(\phi) - \frac{r_1 r_2}{T_1 T_2} \hat{\mathbf{v}}_1(\phi) \cdot \hat{\mathbf{v}}_2(\phi) \right] \right\} \\ & \hat{\mathbf{v}}_1(\pi/2) = -\hat{\mathbf{x}} \cos \phi_0 \cos \beta - \hat{\mathbf{y}} \sin \phi_0 - \hat{\mathbf{z}} \cos \phi_0 \sin \beta \\ & d(\phi = \pi/2) = |\bar{\mathbf{b}} + \bar{\ell}(\pi/2)|; \quad \hat{\mathbf{v}}_2(\pi/2) = -\hat{\mathbf{x}} \\ & \frac{2G(m_1 + m_2)}{d(\phi = \pi/2)} - \frac{G(m_1 + m_2)}{b} = \frac{2G(m_1 + m_2)}{|\bar{\mathbf{b}} + \bar{\ell}(\pi/2)|} - \frac{G(m_1 + m_2)}{b} \\ & = \frac{2G(m_1 + m_2)}{|\bar{\mathbf{b}} + \bar{\ell}(\pi/2)|} - \frac{G(m_1 + m_2)}{\sqrt{r_1^2 - 2r_1r_2 \cos \phi_0 + r_2^2}} \quad (95) \\ & = 4\pi^2 \left\{ \frac{r_1^2}{T_1^2} - 2 \frac{r_1 r_2}{T_1 T_2} \cos \phi_0 \cos \beta + \frac{r_2^2}{T_2^2} + 2 \left[\frac{r_1}{T_1} \hat{\mathbf{v}}_1(\phi) \cdot \left(\frac{d\bar{\ell}}{d\phi} \right) \right. \right. \\ & \quad \left. \left. - \frac{r_2}{T_1 T_2} \left(\frac{d\bar{\ell}}{d\phi} \right) \cdot \hat{\mathbf{v}}_2(\phi) \right] + \frac{1}{T_1^2} \left(\frac{d\bar{\ell}}{d\phi} \right) \cdot \left(\frac{d\bar{\ell}}{d\phi} \right) \right\} \\ & = v_1^2 - 2v_1 v_2 \cos \phi_0 \cos \beta + v_2^2 + 2\bar{v}_\ell(\phi) \cdot [\bar{v}_1(\phi) - \bar{v}_2(\phi)] + v_\ell^2(\phi) \end{aligned}$$

Considering $(T_1 = T_2 = T)$ (62) for the Sun-Earth system, and utilizing the square of the semi-minor axis $(a^2 = r_1^2 - 2r_1r_2 \cos \beta \cos \phi_0 + r_2^2)$ (18) for the Sun-Earth system, we obtain the identity in (96). Although the identity in (95) can be utilized for cases when $(T_1 \neq T_2)$ as well, the result in (96) cannot be directly extended to all planets, because they do not have the same periods with the Sun as Earth does. Hence, according to our circular orbital model for all bodies,

Kepler's 3rd law stating "squares of the orbital periods of the planets are directly proportional to the cubes of the semi-major axes of their orbits" is a very idealistic case which may not apply for most cases. Further, our individual circular orbit model suggests that celestial bodies do not revolve around each other, but rather have relative elliptical orbits observed as a result of their individual circular orbits of revolution, so that the Earth and other planets do not directly revolve around the Sun or vice versa, and the Sun is not stationary in space either.

$$\begin{aligned} & \frac{2G(m_1 + m_2)}{|\bar{\mathbf{b}} + \bar{\ell}(\pi/2)|} - \frac{G(m_1 + m_2)}{b} \\ &= \frac{4\pi^2 (r_1^2 - 2r_1 r_2 \cos \beta \cos \phi_0 + r_2^2)}{T^2} + 2\bar{v}_\ell(\phi) \cdot [\bar{\mathbf{v}}_1(\phi) - \bar{\mathbf{v}}_2(\phi)] + v_\ell^2(\phi) \quad (96) \\ &= \frac{4\pi^2 a^2}{T^2} + 2\bar{v}_\ell(\phi) \cdot [\bar{\mathbf{v}}_1(\phi) - \bar{\mathbf{v}}_2(\phi)] + v_\ell^2(\phi) \end{aligned}$$

As a result of our analysis and assertions, Kepler's three laws of planetary motion appear to be a limited relative interpretation that manifests itself as a special case of actual orbital mechanics, which can be expressed in a broader perspective with our individual circular orbit model.

Data

Data created and analyzed in this study is shared in the Reference [3] data file.

Author Contributions

Aslı Pinar Tan is the only author contributing to this article.

About the Author

Aslı Pinar Tan's findings in this Article are part of all her findings as a result of her personal theoretical studies and research over the years independent from any institution or university, which she has published in the book "Everything Is A Circle: A New Model For Orbits Of Bodies In The Universe" [1], and will further publish in a set of books and other articles. She is an Electrical & Electronics Engineer, Bsc. 1995 and Msc. 1997 graduated from Bilkent University, Ankara, Turkey, and she also holds a degree in Multi-Disciplinary Space Studies program graduated in 2000 from International Space University, Strasbourg, France. She has worked professionally more than 25 years in the Global ICT/Telecom industry, in addition to her academic career in Electromagnetics and Space Applications, all independent from this research. She has been granted a Young Scientist Award by URSI (Union Radio Science Internationale) in 1998 based on her Master Thesis Research results presenting a systematic methodology for High-Frequency Radar Antenna Design using the Genetic Algorithm and Equivalent Edge Currents, to generate plane waves in the near field, which is utilized in the industry since then. (<http://www.linkedin.com/in/aslipinartan>)

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

- [1] Haug, E.G. (2023) Not Relying on the Newton Gravitational Constant Gives More Accurate Gravitational Predictions. *Journal of Applied Mathematics and Physics*, **11**, 3124-3158. <https://doi.org/10.4236/jamp.2023.1110205>
- [2] Tan, A.P. (2020) Everything Is a Circle: A New Model for Orbits of Bodies in the Universe. Amazon Kindle Publishing.
<https://www.amazon.com/gp/product/B08NYG14X8>
<https://www.amazon.com/gp/product/B08PVS2FBW>
- [3] Tan, A.P. (2021) Distance between Two Circles in Any Number of Dimensions Is a Vector Ellipse. Preprints. <https://doi.org/10.20944/preprints202104.0632.v1>
- [4] Tan, A.P. (2020) File: Asli Pinar Tan Analysis Based on Earth-Sun Distance (d) Landsat.xlsx.
<https://www.dropbox.com/scl/fi/th5d3d5ur8d1mb60aitou/Asli-Pinar-Tan-Analysis-Based-on-Earth-Sun-distance-d-Landsat.xlsx?dl=0&rlkey=m3spxqxbxtnumrj2wvtg33l2v>
- [5] Logsdon, T. (1998) *Orbital Mechanics: Theory and Applications*. John Wiley & Sons. <https://books.google.com/books?id=C70gQI5ayEAC>
- [6] Lissauer, J.J. and de Pater, I. (2019) *Fundamental Planetary Sciences: Physics, Chemistry, and Habitability*. Cambridge University Press, 29-31.
<https://doi.org/10.1017/9781108304061>
- [7] Wertz, J.R., Everett, D.F. and Puschell, J.J. (2011) *Space Mission Engineering: The New SMAD*. Space Technology Library, 963-970.
<https://www.amazon.com/Space-Mission-Engineering-Technology-Library/dp/1881883159>